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When, on the other hand, the ratio n of the breadths becomes nearly unity the correction approximates to

$$\frac{(n-1)^2}{\pi}\left\{\frac{1}{2}+\frac{1}{n}\log_e\frac{2}{n-1}\right\},$$

and when it is very nearly unity to

$$\frac{(n-1)^2}{\pi}\left\{1\cdot 19-\log_e(n-1)\right\}.$$

In order to exhibit the character of this correction the following table has been calculated.

Correction to be added to the length of one conductor in terms of the breadth of that conductor, in order to make the total resistance equal to the sum of the resistances of the two conductors, each considered as part of a conductor of infinite length.

Ratio of breadths.	No. of breadths to be added to length.	Ratio of breadths.	No. of breadths to be added to length.
1·0	·0	3·0	·48
1·2	·032	4·0	·65
1·4	·080	6·0	·90
1·7	·17	10·0	1·24
2·0	·25	15·0	1·46
2·5	·37	20·0	1·65

Discussion.

Dr Russell congratulated the Author on having obtained the exact solution of an important problem and thanked him for giving it in a form in which it could be utilised readily by electricians. Somewhat similar problems are of frequent occurrence in practice, in particular he instanced the measurement of the resistance of the bonds connecting the rails in electric tramway systems. The difficulty in this case is in knowing where the rail ends and the bond begins.

A small variation in the position of the potential contacts makes a large variation in the reading of the galvanometer. In many cases the only way of attacking the problem is to calculate the resistance from the known resistivity of the metals by the approximate method indicated by Maxwell. An exact solution, therefore, like the one obtained by the Author, would be of great value in checking the accuracy of the approximate method.

XXIII. *Homogeneous Secondary Röntgen Radiations.* By

Charles G. Barkla, *M.A., D.Sc., Lecturer in Advanced Electricity, and* Charles A. Sadler, *M.Sc., Demonstrator in Physics, University of Liverpool* *.

Though there are many phenomena of Secondary Röntgen Rays still awaiting investigation, it seems desirable in publishing the results of recent experiments—principally on the homogeneous secondary radiations—that a general survey should be made of the whole subject of "Secondary X-Rays emitted by substances subject to X-Rays," and that a more concise statement of the experimental results and the conclusions based on these should be given. This, indeed, appears a necessity not only in order to make intelligible the results of what would otherwise appear isolated experiments of little significance, but to exhibit the observed limitations or the generality of laws which are continually being tested by further experiments on a variety of substances and under a variety of conditions.

As has been shown in previous papers †, the behaviour of substances subject to X-rays varies enormously with the atomic weight of the substance exposed, and generalizations cannot safely be arrived at except after an extensive series of experiments on a large number of elements.

The results which have so far been found to be perfectly general will be briefly stated ‡ :—

* Read June 12, 1908.

The expenses of this research have been partially covered by a Government Grant through the Royal Society.—C. G. B.

† As frequent references are made to the following papers, they are denoted by the letters *a–g* :—

Barkla : *a.* Phil. Mag. June 1903, pp. 685–698.
b. Phil. Mag. May 1904, pp. 543–560.
c. Phil. Trans. A, vol. 204, 1905, pp. 467–479.
d. Roy. Soc. Proc. A, vol. 77, 1906, pp. 247–255.
e. Phil. Mag. June 1906, pp. 812–828.
g. Phil. Mag. Feb. 1908, pp. 288–296.
Barkla & Sadler : *f.* Phil. Mag. Sept. 1907, pp. 408–422.

‡ These results were given in the papers to which reference has already been made ; later experiments have not revealed any exceptions. It appears quite possible, however, that under certain conditions

All substances subject to X-rays are a source of secondary X-rays.

The radiation from a given element is independent of the physical state of the substance and of its mixture or even chemical combination with other elements.

The character of the secondary radiation from an element is independent of the intensity of the primary radiation producing it.

The intensity of secondary radiation from an element is proportional to the intensity of the primary radiation of definite character producing it.

The absorption by a thin sheet of any substance of the secondary rays from various elements subject to the same primary beam is a periodic function of the atomic weight of the radiating substance.

There are, however, groups of elements of neighbouring atomic weight into which substances may conveniently be divided; for when a primary beam of ordinary penetrating power is used, the radiations from the various elements in one group are very similar in properties, while those from elements in different groups differ considerably. But it should be understood that this grouping is somewhat arbitrary, as elements of intermediate atomic weight emit radiations possessing intermediate properties, and the classification depends to a certain extent on the character of the primary radiation. It is, however, convenient for the purpose of description.

H–S Group.

The group of substances of atomic weights from that of hydrogen to that of sulphur appears simplest in behaviour under X-rays of ordinary penetrating power.

Each element, when subject to such a primary beam, emits a secondary radiation which has almost exactly the

exceptions will be found. Crowther, who by careful experiment has further verified some of these results (Phil. Mag. Nov. 1907), finds much less intense radiation from nickel when in combination as nickel carbonyl than we have obtained from the pure element in the solid state. As this cannot be accounted for by variation in the primary beams used, it is perhaps worthy of further investigation.

same penetrating power as the primary producing it. The secondary beam is complex like the primary, and contains the rays of various penetrating powers in approximately the same proportion as the primary (*a* & *b*).

Though it is very difficult, if not impossible, to detect by direct methods a difference in the penetrating powers of primary and secondary beams, when the primary is not more than moderately penetrating, it appears from indirect evidence that the secondary radiation is always slightly more absorbable than the primary (*b*). With more penetrating primary rays the difference is more marked (*b* & *).

The intensity of radiation emitted by these elements is proportional merely to the quantity of matter passed through by a primary beam of definite intensity, if of low to moderate penetrating power: in other words, the intensity of radiation from an atom is proportional to the atomic weight † (*a* & *b*).

The secondary radiation proceeding from one of these substances in a direction perpendicular to that of propagation of the primary is fairly completely polarized, when the rays are of the absorbable type (*d*).

The intensity of secondary radiation from each of these substances varies in different directions perpendicular to that of propagation of a polarized primary beam (*e*).

The amount of polarization in a primary beam, as indicated by the secondary rays, diminishes with an increase in the "hardness" of a given X-ray tube emitting the primary radiation.

The secondary radiation from these substances is approximately twice as intense in the direction opposite to that of propagation of the primary rays as the average in directions at right angles, when the primary beam consists of rays of the easily absorbed type (*g*). (Polarization produces variation in different directions at right angles to the primary beam.)

* Beatty, Phil. Mag. Nov. 1907, pp. 604–614.

† It was considered possible that the discrepancy in the case of hydrogen, as found by one of us, might be explained by the mixture of a small quantity of air. Crowther, however, from more recent experiments has concluded that hydrogen and helium in this group are exceptions to this law of intensity.

This ratio varies somewhat with a variation in the character of the primary rays, but has not been found to exceed 2:1.

The results of all the experiments when an easily absorbed primary was used as the exciting beam, may be explained on the theory as given by Professor J. J. Thomson * shortly after the earliest systematic experiments on light gases. The electric displacement in the primary Röntgen pulses when passing over the electrons produces accelerations in these in the direction of that displacement, and thus causes the emission of secondary pulses of equal thickness. The natural deductions from this theory have all been strikingly verified by experiments on substances of low atomic weight when subject to an easily absorbed primary beam.

Before the phenomena of secondary X-rays from these light atoms may be said to be fully understood, we must explain the effects produced when the primary rays change to those of more penetrating type. In experiments that have been described the secondary rays began to differ in penetrating power from the primary,—they were more easily absorbed; they gave less evidence of polarization of the primary beam, the variation falling from about 20 per cent. to 6 or 7 per cent. in experiments made while the primary became more penetrating; the ratio of intensity of secondary radiation in the direction opposite to that of propagation of the primary beam to that in one at right angles dropped considerably; the ratio of ionization in the secondary electroscope to that in one testing the primary beam increased slightly.

These results might be explained qualitatively either by the introduction of a secondary radiation of different type superposed on the almost perfectly scattered, or by the scattering becoming more imperfect by the introduction of forces of considerable magnitude other than those produced directly by the primary pulses during the passage of those primary pulses over the electrons, or by the introduction of a greater proportion of tertiary rays due to

* 'Conduction of Electricity through Gases' (2nd edition) p. 321.

the emergence of the secondary rays from greater depths of the radiating substance, or by a combination of these.

Though the experiments, the results of which are stated above, were not performed concurrently, it was evident that the variation in intensity of secondary radiation exhibiting the polarity of the primary beam changed from about 20 per cent. to only about 6 or 7 per cent. as the X-ray tube became "harder," even before a difference between the penetrating powers of primary and secondary beams could be detected by direct comparison. It was not evident whether this change was actually one in the polarization of the primary beam itself or in the efficiency of the secondary rays in exhibiting a polarization of constant magnitude. The latter appeared the more probable when considered in conjunction with the changes that had been found in the ratio of intensities of secondary rays in a direction almost opposite to that of primary propagation and one at right angles. Further experiments were therefore made to determine if the changes were all attributable to the same cause. It was found, however, that although increasing the hardness of a given X-ray tube produced a decrease in the amount of polarization detected, the more penetrating portion of a primary which was transmitted through a sheet of aluminium did not exhibit less polarity but slightly more, indicating that the effect was due not merely to change in the penetrating power of the radiation but to some change in the polarity of the primary beam itself. This was supported by the fact that the secondary radiation did not become appreciably different in penetrating power from the primary producing it,—indicating a fairly perfect scattering. Finally, later experiments have shown that for a primary radiation proceeding from a tube in the state of hardness which has invariably been found to exhibit a minimum of polarity in the primary, the ratio of intensities of secondary rays in the two directions indicated has been such as would be given by an almost perfectly scattered radiation.

We cannot then attribute the decrease in the amount of polarization of a primary beam exhibited by the secondary rays to the scattered rays being only partially a scattered radiation or to imperfection of scattering, but it is almost

certainly a decrease in the polarity actually existing in a primary beam when the tube becomes harder.

These results are possibly due to the more swiftly moving cathode particles in the X-ray tube being productive of more secondary cathode rays in the anti-cathode. As the secondary cathode particles are not directed like the primary cathode rays, they produce radiation which is not polarized. The greater the number of secondary cathode particles produced, the less is the polarity of the complex radiation. As the X-radiation from the secondary cathode particles is probably less penetrating than that due to the primary cathode particles, the more penetrating portion of the complex X-radiation exhibits slightly more polarity than the more easily absorbed.

The small increase observed in the intensity of secondary radiation from air, paper, &c., as measured by the ionization produced in an electroscope, when the primary beam is made more penetrating, is possibly due to the superposition of a homogeneous unscattered radiation, such as is emitted by elements of higher atomic weight. This would account for the complex secondary radiation differing more and more in penetrating power from the primary as the latter became more penetrating; for it has a definite penetrating power characteristic merely of the element emitting it. Though experiments have not yet been made to analyse this secondary radiation set up by the more penetrating radiation, it appears, for reasons discussed later, exceedingly probable that such a radiation does appear when a penetrating primary beam is used.

A point still awaiting investigation is the change in the observed ratio of intensities of secondary radiation in directions approximately opposite and perpendicular to that of primary propagation. Experiments have been made to ascertain the amount by which this ratio is affected by a change in the polarization of the primary beam, by the superposition of tertiary rays in greater proportion, and by the superposition of homogeneous radiation characteristic of the radiating element. In these later experiments, however, the deviation from the theoretical ratio for perfect scattering has through all the changes made in the primary

beam been much less than in the first experiments ; and it has been found that even a fairly penetrating primary beam—much more penetrating than any used in the experiments referred to in a previous paper (*g*)—sets up secondary rays whose intensity distribution is within a few per cent. of that which would be given by perfect scattering. This matter is being further investigated *.

Cr–Zn Group.

The radiation proceeding from elements of atomic weight between those of chromium and zinc, when subject to X-rays of ordinary penetrating power, is of a very different type from that discussed, for from no two elements in this group is the penetrating power the same. The absorption by a thin sheet of aluminium ·0104 cm. thick is, between these limits of atomic weight, a decreasing function of the atomic weight. varying from 94 per cent. for the radiation from chromium to 64 per cent. for the radiation from zinc. [One of the primary beams used was absorbed to the extent of about 34 per cent.]

Homogeneity.—One of the most remarkable features about the radiation from any one of these elements is that though the primary rays incident upon the substance are very heterogeneous, that is consist of rays varying considerably in penetrating power, the secondary rays are homogeneous. This point has been briefly referred to in a previous paper (*f*).

To give a particular example :—The ionization produced in a given electroscope by a primary X-ray beam was diminished by 51 per cent. by placing a sheet of aluminium ·0208 cm. in thickness in its path ; after 77 per cent. had been absorbed by aluminium, a similar plate produced a

* It ought to be remarked, that though an elementary consideration of the production of secondary rays indicates that the intensity of radiation is the same in the forward and backward directions, and that each of these is double that in a direction at right angles, a more complete theory shows that these results can at best be only approximately true. It is only necessary to consider the action of the magnetic field in the primary pulses on the electrons as they begin to move under the action of purely electric forces, to see that dissymmetry must exist. A complete theory must also take into consideration the distribution of the tubes of electric force round each radiating electron.

further diminution of this ionization by 27 per cent.; after 91 per cent. had been absorbed the same plate cut off only 18 per cent., showing that the rays after each transmission became on the average more and more penetrating. This effect has been explained as due to the more absorbable constituents being sifted out.

Although such a primary beam produced in one of this class of substances the secondary radiation experimented upon, it was found that the secondary radiation was of an entirely different type, being equally absorbed after transmission through sheet after sheet of absorbing substance.

The radiation from thick copper was found not to differ appreciably from that from a very thin sheet which was only thick enough to absorb 14 per cent. of the primary rays. Thus the radiations from the deeper layers after transmission through the surface layers were of the same character as those from the surface layers. Consequently, in dealing with these secondary rays it is not necessary as in the H—S group to deal with very thin plates in order to determine the character of the radiation as emitted by the atoms themselves.

Using zinc as the radiating substance, the direct ionizing effect of the secondary rays was determined, and afterward the ionization produced by the same beam after transmission through thin sheets of zinc and aluminium placed at a distance of several centimetres from the detecting electroscope in order to avoid complications due to the more easily absorbed corpuscular secondary rays from the metal sheet.

The effect of the radiation from air was determined by separate experiments, and correction was then made for this in each observation, though when a large proportion of the secondary rays was absorbed, the air radiation was quite a considerable fraction of the whole and the possible error was as a consequence greatly increased.

Below are tabulated the percentage absorptions by a plate of zinc ·00131 cm. thick and one of aluminium ·0104 cm. thick of the secondary beam direct from zinc and of this beam after transmission through various thicknesses of aluminium. These results exhibit the striking homogeneity of the radiation from zinc.

TABLE I.

Radiation from Zinc (thick sheet).

I. Percentage Absorption by Al previous to absorption in column II. or III.	II. Percentage Absorption by Zn (·00131 cm.) after absorption in column I.	III. Percentage Absorption by Al (·0104 cm.) after absorption in column I.
0	*36·5*	*67·5*
22	36·2	67
67	35·4	—
0	*35·8*	—
89	35·4	67
97	33·9	66
0	*34·2*	—

Similar experiments were made on the absorption of the radiation from zinc by zinc when that radiation had been passed through various thicknesses of zinc to absorb different proportions of it.

The radiation from copper was examined in the same way.

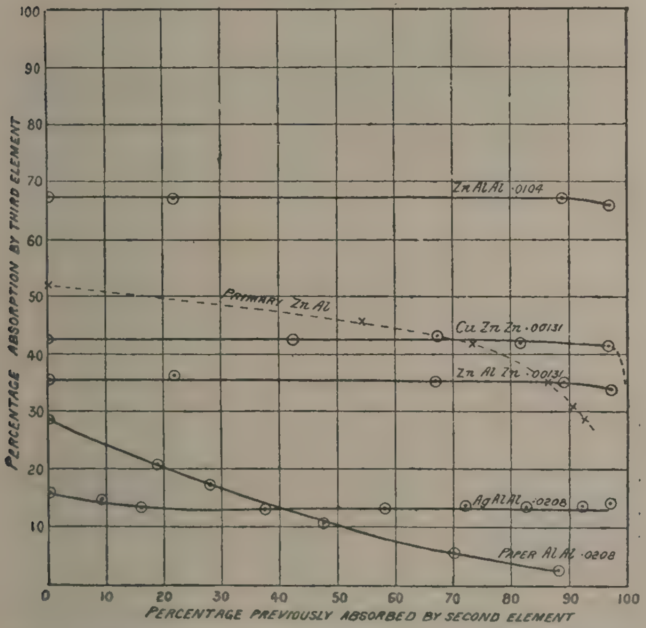
TABLE II.

Radiation from Copper (thick sheet).

I. Percentage Absorption by Zn previous to absorption in column II.	II. Percentage Absorption by Zn (·00131 cm.) after absorption in column I.
0	*43*
44	44
0	*41·4*
42·6	42·6
67·5	43·1
81·5	42
96·7	41·8
0	*42·5*

All these experiments show extremely little change in the percentage absorption even after almost complete absorption. The contrast between primary and secondary beams is strikingly shown in fig. 1, in which the absorptions are represented by ordinates and the amount previously absorbed by abscissæ. The corresponding curve for the secondary radiation from paper (in this case subject to penetrating primary rays) is given for comparison (fig. 1).

Fig. 1.



Note.—Cu Zn Zn ·00131 indicates:—Copper radiation after transmission through Zn is absorbed by Zn ·00131 cm. thick to extent shown by ordinates.

Independence of Primary Rays.—To exhibit the independence of the penetrating power of the secondary radiation from one of these metals Cu, of that of the primary producing it, we have tabulated below the absorbability of various primary rays and that of the secondary rays produced

by these. Though the absorption of the primary by aluminium ·0208 cm. thick varied from 52 to 18 per cent., the absorptions of the corresponding secondary beams from copper by aluminium ·0104 cm. thick were as nearly as observable the same, the experimentally determined values being 58·3 and 58·1 per cent. respectively. This constancy in character makes accurate experiments on these radiations possible.

TABLE III.

Radiation from Copper (thick sheet).

I. Percentage Absorption by Al of Primary Radiation previous to incidence on Cu radiator.	II. Percentage Absorption of Primary Rays by Al (·0208 cm.) after absorption given in column I.	III. Percentage Absorption of Secondary Rays by Al (·0104 cm.).
0	..	58
52	..	58·6
0	52	58·3
77	27	58·8
0	51	58·4
91	18	58·1
0	51	58·8

Each of these elements therefore emits a characteristic radiation which is independent of the penetrating power of the primary beam setting up this radiation, and being homogeneous, the absorption-coefficient calculated from the relation $I = I_0 e^{-\lambda x}$ has a value which is independent of the thickness x of absorbing substance *—a property which appears to be unknown among X-ray beams hitherto experimented upon.

The absorption-coefficients for the radiations from Fe,

* After transmission through sheets of absorbing substance, secondary rays are superposed on the transmitted radiation, and the resultant radiation in some cases ceases to be even approximately homogeneous. This effect can, however, be readily distinguished from true heterogeneity by using as absorbers only those substances in which a radiation of different type is not stimulated.

Co, Ni, Cu, and Zn, when absorbed by Al, Fe, Cu, Zn, Ag, Sn, and Pt have been given in a previous paper (*f*).

Though it has been shown that the secondary radiation from some of the substances in this group (Cr–Zn) is remarkably homogeneous in comparison with the primary which produces it, the test applied is not one of extreme delicacy, and the presence of the scattered radiation similar to that of the first group (H–S) and of intensity given by the law found for that group would be exceedingly difficult to detect, as it would produce only about $\frac{1}{300}$ of the total ionization actually produced by the secondary rays from a very thin sheet of copper. Now we have seen that a radiation after transmission through metals may contain a considerable proportion of the radiation which is characteristic of the metal traversed and which was not in the incident radiation. This acquired radiation may even, if the substance

TABLE IV.

Radiation from Copper (thick sheet).

I. Percentage Absorption by Cu previous to absorption by Cu, Zn, or Al.	II. Percentage Absorption by Cu, Zn, and Al after absorption in column I.		
	Cu ·00296 cm.	Zn ·00262 cm.	Al ·0104 cm.
0	74	69·7	72
98·3	70	66	68

Radiation from Iron (thick sheet).

I. Percentage Absorption by Fe previous to absorption by Fe, Cu, or Al.	II. Percentage Absorption by Fe, Cu, and Al after absorption in column I.		
	Fe ·00313 cm.	Cu ·00296 cm.	Al ·0104 cm.
0	80·5	91·5	90
80	79·8	91	89·5
96	76·9	86	83·5

traversed has almost completely absorbed the incident radiation, constitute the bulk of the transmitted radiation. In such a case, experiments on the transmitted beam in order to analyse its constituents would be misleading, the constituents not being present in the original beam.

To eliminate the possibility of this error, we have tested the radiation from copper after transmission through thin sheets of copper by absorbing by further plates of copper. Thus radiation of a fresh type was not introduced.

It was found that after absorbing 98·3 per cent. of the copper radiation by copper, the absorption by copper ·00296 cm. thick had dropped from 74 to 70 per cent. The effect was more pronounced in the case of the radiation from iron, the numerical data for which are given below.

There was thus evidence of a slight heterogeneity even in these radiations.

Scattered Rays.—To test if this heterogeneity could be accounted for by the mixture of a scattered radiation, like that from light atoms, with the homogeneous radiation, a direct comparison was made between the ionization produced by the secondary beams from thin sheets of copper [·00067 cm.] and paper, subject to the same primary radiation. The paper which was used had ten times the mass of the thin copper, yet the ionization produced by the secondary rays from the copper was 19·5 times that produced by the secondary rays from paper even though a greater proportion of the radiation from copper was absorbed by the copper itself than that from the paper in the paper. As sheet after sheet of aluminium was placed in the path of the two secondary beams, the copper radiation was absorbed to a much greater extent than the radiation from paper; thus 34 per cent. of the radiation from paper was transmitted through aluminium ·0416 cm. in thickness, while only ·7 per cent. of the copper radiation got through. The ratio of the ionization due to the copper radiation to that due to the rays scattered from paper, after transmission through ·0416 cm. Al had dropped to ·401 : 1. After transmission through ·0782 cm. of Al the two radiations appeared approximately equal in penetrating power, the last ·26 per cent. of the

copper radiation being much more penetrating than the homogeneous radiation which had been practically all absorbed. When the two transmitted radiations were approximately of the same character, the ratio of their ionizing effects was about ·18 : 1 ; or from equal masses of copper and paper—disregarding all internal absorptions the intensities of the penetrating rays were in the ratio 1·8 : 1.

TABLE V.

I. Thickness of Al in path of Secondary beams.	II. Percentage of Cu radiation absorbed.	III. Percentage of Paper radiation absorbed.	IV. Ratio of ionizations due to Secondary beams from equal masses of Cu and Paper.
0	0	0	195·5 : 1
·0208	94·5	51·8	22·1 : 1
·0416	99·3	66	4·01 : 1
·0574	99·6	71	1·94 : 1
·0782	99·74	78	1·88 : 1
·0990	99·8	83	1·83 : 1

Allowing for the small extra absorption of the penetrating portion of the copper radiation in the copper plate itself above that of the corresponding rays in the paper—quantities which were determined by separate experiments—the radia tion from copper must have contained approximately twice as much of the penetrating radiation as the radiation from paper.

By using a thicker copper plate as radiator the intensity of secondary radiation was increased, but the correction for absorption in the metal itself was also increased so that the result could not be regarded as more accurate, this correction not being obtainable with great accuracy. The conclusion was however practically identical with the above. We thus see that mixed with the homogeneous radiation from copper is a more penetrating radiation. The most penetrating portion of this is about twice as intense as the corresponding radiation from substances of the H–S group.

The radiations from other elements of the Cr–Zn group have not been examined so minutely as that from copper, but it has been seen that these also contain a small quantity of a more penetrating radiation which is probably scattered radiation. This more penetrating radiation was more evident in the radiation from thick iron, probably because the homogeneous iron radiation being very absorbable emerges from a thinner surface layer, while a scattered radiation emerges from greater depths than in the metals of higher atomic weight owing to the greater transparency of iron than substances of higher atomic weight. Consequently the scattered rays—if we may assume them to be such—are in reality from a much greater mass of iron than the homogeneous rays, and produce more than their appropriate portion of the total ionization.

It may be objected that in this case the radiation was transmitted through a second substance Al, and may have contained a considerable quantity of secondary radiation from Al of a kind not existent in the original radiation from copper. The production of a secondary radiation more penetrating than the primary producing it is, however, contrary to all experience.

Energy.—The ionization produced by the secondary rays from one of the elements of this group has been shown to be enormous in comparison with that produced by the secondary scattered rays from an equal mass of an element of low atomic weight. From a sheet of copper ·00067 cm. thick, absorbing 14 per cent. of the primary radiation, the secondary radiation produced an ionization in the detecting electroscope 200 times as great as that from an equal mass of paper. Correcting for absorption of primary and secondary rays, the ratio of ionization produced by the rays from equal masses of copper and paper was approximately 300 : 1. This is considerably greater than would have been found if all the radiation absorbed had been simply scattered as an untransformed radiation. This, however, by no means gives us a measure of the energy of the secondary rays, for these are of much more absorbable type than the primary. The percentage absorption by a thin sheet of aluminium is about five times as great for these secondary rays as for the primary.

If we assumed the same ratio for the ionizations produced in air by the two radiations if of equal intensity, we should be led to conclude that the energy of this homogeneous radiation is about 45 times that of the scattered radiation from an equal mass of paper, and about $\frac{1}{3}$ the total energy absorbed in the copper *.

Though it is impossible by such experiments to determine the energy with accuracy, we may safely conclude that the energy of the homogeneous radiation is many times greater than the energy of secondary radiation scattered from an equal mass of one of the light elements.

Comparisons of the ionizations produced by the rays from other elements of this group have been made. They are all of the same order of magnitude.

Distribution.—It has been shown by one of us that the secondary radiation from thick copper, when this is subject to a primary beam of ordinary penetrating power, is approximately equally intense in a direction almost opposite to that of propagation of the primary and in a direction at right angles. As probably 98 per cent. of the ionization produced by the secondary radiation from thick copper is due to the homogeneous rays, this may be said to be the result for the homogeneous rays alone.

From thick iron, however, the radiation varied in intensity by an amount represented by the ratio 1·1 : 1 in these two directions. But as we have shown, the heterogeneity of the radiation from thick sheets of iron is more marked, and this can be accounted for by the fact that from iron the homogeneous rays are very easily absorbed, consequently scattered rays emerge from a much thicker layer and appear in more than their normal proportion. The ratio 1·1 : 1 verifies this by showing the presence of a radiation which is controlled by the electric field in the primary pulses.

The result is of the order of magnitude that would be given by a mixture of scattered rays of about the same

* The homogeneous radiation has been assumed to be distributed uniformly in all directions and the scattered radiation to be proportional to $\sin^2 \theta$, where θ is the angle between the direction of radiation considered and that of acceleration of the radiating electron.

intensity as found in the radiation from copper with the characteristic radiation uniformly distributed.

Polarization Experiments.—Though very careful experiments have been made with iron, copper, and zinc as secondary radiators placed in a partially polarized primary beam of Röntgen radiation, the secondary rays from these have not been found to give evidence of any polarity. Thus the intensity of secondary radiation in a given direction is independent of the position of the plane of polarization of the primary beam producing the radiation; in other words, the intensity of secondary radiation from members of this group is independent of the direction of electric force in the primary radiation. Again, this result may be taken as applicable to the homogeneous rays, as these constitute the bulk of the radiation from these metals.

Efficiency of Primary Rays as Secondary Ray Producers.—Although in addition to the relatively small amount of scattered radiation, rays of only one penetrating power were emitted by an elementary substance upon which a heterogeneous primary beam fell, it was still possible that only one constituent (rays of one penetrating power) in each heterogeneous primary beam was producing this radiation. It was therefore important to determine to what extent each constituent of the primary beam was effective in producing these secondary rays.

To do this, a portion of the primary beam direct from the X-ray tube was sent through one electroscope while another portion was incident on a secondary radiator, some of the rays from which passed through a second electroscope. The method was then simply to place absorbing plates in the primary beam before falling on the radiator, and to observe the extent by which the primary and the secondary radiations were reduced. It is obvious that the more penetrating constituents of the primary beam penetrate to greater depths than the absorbable constituents, and so are really transmitted through a greater mass than these. Hence, if two homogeneous constituents of primary radiation in passing through equal masses of radiating substance were equally efficient as secondary-ray producers, the radiation emitted by a thick plate would be produced principally by the more

penetrating constituents; consequently an absorbing plate placed in the position indicated, would, by cutting off the more easily absorbed constituents, produce less diminution of the ionization in electroscope E_2 than of that in E_1. It was therefore necessary to use as the radiator a sheet of metal which would absorb very little of the primary radiation, so that even the deepest layers would transmit different constituents in proportions approximately the same as those transmitted through the first surface-layer.

It was found that a sheet of Cu ·00067 cm. in thickness when placed in the primary beam produced an absorption of only 14·5 per cent., as measured by the ionization produced in an electroscope. This was considered sufficiently thin for use as a radiator. The deflexions of the electroscopes were first observed when no absorbing plate was used. Aluminium

TABLE VI.

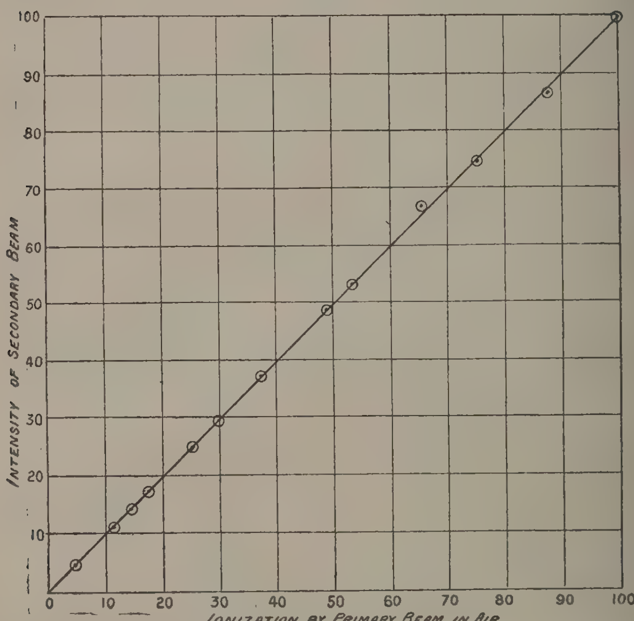
Radiation from Copper (thin sheet absorbing 14·5 per cent. of primary rays.)

I. Thickness of Al in Primary beam in centimetres.	II. Absorption of Primary by Cu (·00293 cm.) after passing through Al in column I.	III. Percentage of Primary absorbed by Al in column I.	IV. Percentage diminution of Secondary by Al in column I. absorbing Primary.	V. Ratio of Ionizations in Secondary and Primary electroscopes (relative).
0	84	0	0	1
·0032	82·8	12·2	13·6	·98
·0064	81·4	24·9	25·5	·99
·0104	80·3	34·2	33·1	1·01
·0168	77·5	46·7	46·6	1·00
·0208	75·9	51·1	51·4	·99
·0312	72·3	62·9	62·9	1·00
·0416	69·6	70·2	70·5	·99
·0520	65·2	74·6	75·0	·98
·0728	60·4	82·4	82·3	1·00
·0936	57·0	85·8	85·8	1·00
·124	51·2	88·6	88·9	·97
·248	41·7	95·4	95·3	1·02

plates of thickness shown in column I of Table VI. were placed in the primary beam, and the deflexions of the primary and secondary electroscopes were diminished by the amount given in columns III and IV.

In fig. 2 the ionization produced by the primary beam in air is indicated by abscissæ and that produced by the corresponding secondary beam from a thin sheet of copper as ordinates, when different portions of the primary beam have been absorbed.

Fig. 2.



As the secondary beam is practically homogeneous we have exhibited the relation between the ionization produced by a primary beam in the air, and its power of producing secondary rays in copper. We are thus led to the conclusion that the intensity of this homogeneous secondary radiation set up in a thin sheet of copper is proportional to the number of ions the primary beam would produce in a thin layer of air.

Thus if two beams of Röntgen rays, which in passing through a thin film of air would produce equal ionizations in that air, be sent through a thin sheet of copper, the intensity of secondary radiation produced on that sheet by one beam will equal that produced by the other, even though one is three or four times as penetrating to copper as the other one. As for such ranges of penetrating power as are possessed by the constituents of such a primary beam, the relative ionizations produced by those constituents in different substances are usually fairly constant, it is highly probable that the intensity of the homogeneous rays from copper is through wide ranges of penetrating power of the primary proportional to the ionization which takes place in the copper, and is independent of the character of the primary producing it.

It does not necessarily follow from this that the process of ionization produces the radiation, but it seems to indicate that the energy of the homogeneous secondary radiation is, for such ranges in penetrating power, proportional to the energy of the primary beam spent in the process of ionization.

When the primary radiation was transmitted through copper or iron before falling on the copper radiator, the ratio of intensity of secondary radiation to ionization produced by the primary beam dropped slightly, showing that the radiation transmitted through these substances was less efficient as a secondary-ray producer in comparison with its power of producing ionization in air. This may have been due to the fact that in transmission through these plates, a gradually increasing quantity of secondary radiation from the plates was superposed on the primary. This being in one case of the same penetrating power as the secondary emitted by the radiator, and in the other more easily absorbed than it, would not afterwards produce secondary rays in the copper (see later). When the radiation was absorbed by zinc, the ratio remained fairly constant. This result would, on the corresponding theory, be due to the fact that the radiation from zinc is slightly more penetrating than that from copper, consequently the zinc radiation acquired in the primary beam would be capable of stimulating a feeble secondary in copper. (See Table VII. p. 356.)

TABLE VII.

Radiation from Copper (thin sheet absorbing 14·5 per cent. of primary rays).

I. Thickness of plate in Primary beam.	II. Percentage of Primary absorbed by Zn in column 1.	III. Percentage diminution of Secondary by Zn in column 1 absorbing Primary.	IV. Ratio of Ionizations in Secondary and Primary electroscopes.
Zn ·00131 cm.	59·4	59·6	·99
·00262 „	77·5	77·6	·99
·00393 „	84·8	84·8	1·00
·00524 „	89·1	90·1	·91
·00786 „	93·5	93·6	·98
·01179 „	97·2	96·3	1·32
Cu ·00067 cm.	43·5	45·1	·97
·00134 „	65·0	66·5	·95
·00296 „	82	84·8	·84
·00592 „	90	91·6	·84
·01184 „	97	98	·66
Fe ·00315 cm.	77·2	80·8	·84
·00630 „	89·0	91·4	·78
·00945 „	93·0	95	·71

Whatever may be the true explanation, these results show that the proportionality exhibited after absorption by aluminium is not general even when copper is the radiating substance. This result may be contrasted with the corresponding phenomena exhibited by the homogeneous radiation from silver. In these experiments the homogeneous rays disappeared when the primary beam was made more absorbable and more efficient as an ionizer of air. The difference is almost certainly due to the fact that in the latter case the primary radiation passes from one more penetrating to one more easily absorbed than the secondary radiation characteristic of the element exposed to the primary rays. It seems highly probable therefore, that with sufficiently absorbable primary rays no such relation as that shown by fig. 2 would be obtained, but that the secondary would disappear while the primary still produced considerable ionization in air. It will be interesting to learn if a primary which ceases to stimulate the homogeneous radiation ceases also to produce ionization in the radiating substance. We, of course, know that elements of the H–S group are ionized, but do not

emit a homogeneous radiation which can be detected. This appears to indicate no necessary connexion between ionization and radiation, but is not conclusive for various reasons.

Special Penetrating Power.—In studying the absorption of these homogeneous radiations by a number of elements, it was found that the relation between the absorption by a given substance of the various homogeneous radiations from elementary substances and the atomic weight of those radiating substances was similar for all absorbing substances, except in the case of the radiating and absorbing elements being identical or possessing neighbouring atomic weights. Each substance appeared to be especially transparent to its own radiation, and to a less extent to that from elements of neighbouring atomic weight.

The transparency of various elements to an ordinary heterogeneous beam of X-rays has been investigated by Benoist, who studied the phosphorescence produced by the beam after passing through absorbing substances. He compared the transparencies by finding the mass of a prism of absorbing substance of definite cross-section, which when placed in the path of the beam absorbed it by a definite amount. The relation between the transparency so defined and measured, and the atomic weight of the absorbing substance was shown by a curve similar to that for paper given in fig. 3 (p. 358). It shows a rapid decline of transparency, with increase in atomic weight for low atomic weights, the rapidity of the fall of transparency diminishing with an increase of atomic weight. In the case of very soft rays this decline becomes a slight incline.

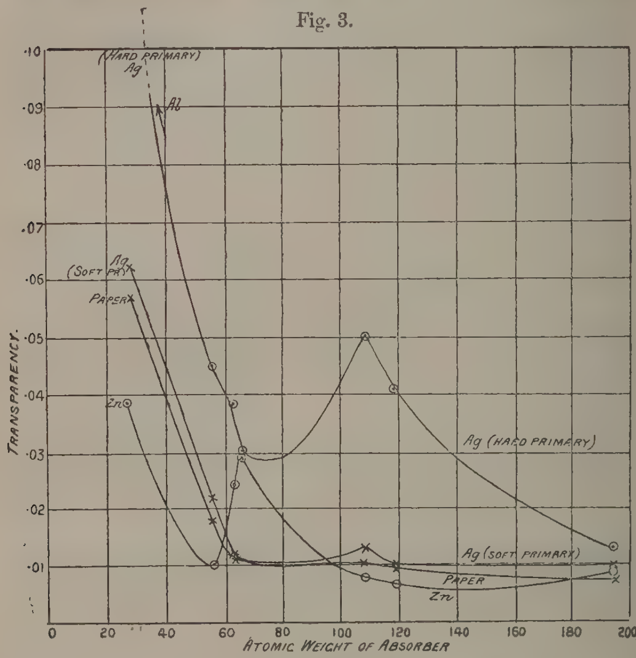
By using thin plates of absorbing elements and testing by the ionization method the percentage diminution of intensity of the secondary beams by transmission through these plates, it was easy to calculate the thickness and hence the mass per unit cross-section necessary to absorb a given proportion —in this case 75 per cent. Some of these results we have given in a previous paper (f) *. They show that in place of the usual relation between transparency and atomic weight there is a strongly marked deviation in each case in the

* In the paper referred to the numbers given in Table II. represent mass in grammes, not thickness in centimetres as stated.

neighbourhood of the atomic weight of the radiator, the rays from an element being especially penetrating to that element and to a less extent to elements of neighbouring atomic weight.

That this special penetrating power is not due to the constituent easily absorbed by the radiating substance having been sifted out before emergence through the surface layer

Fig. 3.



might be inferred from the fact of its homogeneous character, unless of all the constituents in the radiation as emitted from the atom itself this was the only one transmitted through even fairly thin layers of the substance. But we find other secondary rays transmitted with almost the same facility. We may conclude, therefore, that it is not merely from the surface of relatively thick sheets that the radiation emerges in a homogeneous state, but that it is so emitted by the atoms themselves. The special power of penetration is thus a specific property of the secondary rays and does not appear to be due to previous selective absorption.

Tertiary Rays.—Some of the most interesting phenomena in connexion with these homogeneous rays were those shown by experiments made in order to investigate the special penetrating power of these rays.

When the characteristic homogeneous radiation from iron was passed through a thin sheet of copper—a substance whose characteristic radiation is of more penetrating type—it was partially absorbed, and the transmitted radiation appeared unchanged in character.

When, however, the radiation from copper was passed through iron there was superposed on the copper radiation a considerable quantity of iron radiation; when the thickness of the absorbing plate of iron was sufficient to produce almost complete absorption, the bulk of the transmitted radiation was iron radiation.

Similar experiments were made on copper and zinc. The radiation from the former, which is more absorbable than that from the latter, when passed through zinc was transmitted without admixture of other radiations, but when the radiation from zinc was transmitted through copper it became more like the copper radiation. The effect was not so well marked as in the case of the transmission of copper radiation through iron.

Thus a characteristic homogeneous radiation was emitted by a metal when the primary beam to which the metal was exposed was of more penetrating type than the characteristic radiation. When the primary was of a more absorbable type, this characteristic secondary radiation was not emitted*.

Ag–I Group.

When subject to primary beams of moderate penetrating power such as those used in these experiments, the elements of atomic weights from that of silver to that of iodine are exceedingly susceptible to small changes in the penetrating power of that primary radiation, both as regards the character and the intensity of the secondary rays that they emit. This is indeed the most remarkable feature of this group of elements.

Silver and tin are the only elements of this group which have been examined in any detail, but they appear from

* Details of these experiments will be given later.

superficial observation to be typical of the whole group. In previous papers it has been shown that the radiation from these two when subject to a primary beam of only moderate penetrating power, is not a scattered radiation like that which proceeds from elements of the H–S group, for it differs much more in penetrating power from the primary producing it than the radiation from the elements of that group, though not so much as the radiation from Cr, Fe, Cu, &c. The radiation does not exhibit the polarity of a primary beam where such exists, and it is not distributed in the manner of the radiation from substances of low atomic weight, the intensity in a direction almost opposite to that of primary propagation being approximately equal to that in a direction at right angles. It was not possible, however, to perform this experiment with anything like the degree of accuracy with which it was done in the case of the radiation from Fe, Cu, &c., because the ionization produced by the secondary beams from Ag and Sn was much less intense.

Secondary Scattered Rays.—Experiments with the softest rays procurable from an X-ray tube of ordinary type, however, considerably simplified the secondary radiation, for it was found that the secondary rays from silver then differed very little in penetrating power from the soft primary. When the intensity of secondary radiation set up by a powerfully polarized primary was examined in the two principal directions at right angles, it was found that it varied by approximately the same amount as that from elements of the H–S group, thus exhibiting the same perfection of scattering. When tin was examined in the same way, the variation in intensity was found to be about half that exhibited by the rays from paper and from silver. Thus by using a very soft primary radiation, an almost purely scattered secondary radiation was emitted by silver, and a radiation consisting of a large proportion of scattered rays was emitted from Sn.

To compare the intensity of this scattered radiation from silver with that from elements in the H–S group, the ionization produced by the secondary radiation from a thin sheet of silver ·00064 cm. thick was compared with that produced by the radiation from paper. The masses of silver and paper emitting the secondary rays were ·2695 gr. and

2·72 gr. respectively. The relative ionizations produced by the secondary rays from these were 148 and 280. From other experiments on the absorption of the primary beam by sheets of silver and paper of different thicknesses, it was estimated that about 60 per cent. of the silver radiation and 75 per cent. of the paper radiation was transmitted through the surface layer. The intensities of radiation from these masses were, therefore, in the ratio 37 : 56 approximately. From equal masses this was 37 : 5·6, or 6·5 : 1 approximately.

As the primary radiation incident on silver became more penetrating, the ionization produced by the secondary rays increased enormously, the variation of intensity in the two principal directions at right angles to that of propagation of the primary rays gradually decreased from its original amount—about 14 per cent.—until it was inappreciable, though with the same primary the variation in the intensity of radiation from carbon had only dropped to about 6·5 per cent. The evidence of polarity of the primary given by the secondary rays from silver thus disappeared, while that given by the rays from carbon simply decreased from 14 per cent. to 6·5 per cent. It should, however, be noticed that the ionizing effect of the secondary rays from silver increased considerably, and when the evidence of polarity disappeared the total secondary ionization had increased about tenfold, so that such an effect would have been produced by the superposition of the homogeneous radiation, like that emitted by Cu, Fe, Zn, &c.; for such a radiation gives no evidence of polarity in a primary beam, being uniformly distributed around that beam *.

* An early experiment by one of us on the radiation from tin (*e*) indicated that the scattered radiation was not emitted in even the intensity that could be given by an element of the H S group. The experiment was performed by comparing a very penetrating portion of the radiation from tin with that from paper. There are possibilities which make such an experiment inconclusive, and we feel that it requires verification. If the numerical values for the polarization given in Table VIII. (p. 362) could be taken as strictly accurate throughout, we should be led to conclude that the evidence of polarity disappeared more rapidly than could be accounted for by mere superposition of the homogeneous radiation, but the measurements in the final stages were too uncertain for such a conclusion to be based on them. We have, therefore, no conclusive evidence of a disappearance of the scattered radiation.

The evidence of polarity of the primary given by the radiation from tin disappears for even softer primary rays than in the case of silver. This must be connected with the fact that the characteristic homogeneous radiation from tin is less penetrating than that from silver, and is set up by a less penetrating primary radiation. It thus appears earlier in the process of hardening the primary and swamps the effect of the scattered radiation sooner.

TABLE VIII.

I. Absorption of Primary by ·01 Al.	II. Ratio of Ionizations in Secondary and Primary electroscopes.		III. Percentage Variation of intensity of Secondary radiation exhibiting polarity of Primary beam.	
	C radiator.	Ag radiator.	C radiator.	Ag radiator.
35·5	·303	·180	14	11·5
32·6	·340	·359	10 95	4·45
32·3	·318	·305	12 9	6·2
32·0	·315	·369	12·9	4·5
30	·349	·821	9·9	2·8
29·4	·353	·844	9·6	2·45
28·6	·364	1·12	9·15	·9
28·9	·380	1·35	7·5	·75
27·3	·405	1·62	7	·45
25 6	·425	1·82	6·5	0

Homogeneous Rays.—An analysis of the radiation from silver similar to that made of the radiation from elements of the Cr–Zn group shows that when the primary radiation is moderately penetrating, such a homogeneous secondary radiation constitutes the bulk of the rays emitted. Absorption by thin sheets of aluminium showed slight heterogeneity at first, such as would be evident if the scattered radiation were superposed on the more penetrating homogeneous radiation. After the absorption of this more easily absorbed scattered radiation the remainder appeared perfectly homogeneous.

The contrast between the constitution of the copper and silver radiations is shown by the curves given in fig. 1 (p. 345).

In the copper radiation the homogeneous rays are more easily absorbed than the scattered rays, so the curve is initially horizontal and finally slopes downwards. In the silver radiation the scattered rays are on the average more easily absorbed than the homogeneous rays, consequently the curve dips initially and finally becomes horizontal. The absorptions at successive states are shown in the following tables. With these results may also be contrasted those obtained for the scattered radiation from paper.

TABLE IX.—Radiation from Silver (thick).

Thickness of Al placed in Secondary beam from Ag.	Percentage diminution of Ionization produced by Secondary rays by transmission through Al in column 1.	Percentage diminution of Ionization by Secondary rays due to absorption by further sheet of Al (·0208 cm.).
0	0	15·9
·0104 cm.	9·2	14·8
·0208 ,,	16	13 8
·0574 ,,	37·7	13·1
·124 ,,	58	13·5
·182 ,,	72	14
·263 ,,	82·5	13·6
·387 ,,	92	13·8
·526 ,,	97	14·2
0	0	16

TABLE X.—Radiation from Paper.

Thickness of Al placed in Secondary beam from Paper.	Percentage diminution of Ionization produced by Secondary rays by transmission through Al in column 1.	Percentage diminution of Ionization by Secondary rays due to absorption by further sheet of Al (·0208 cm.).
0	0	28·1
·0104 cm.	18·7	20·9
·0208 ,,	28	17·4
·0574 ,,	47·5	10·9
·182 ,,	70	5·8
·584 ,,	88	2·5
0	0	27·4

W–Bi Group.

The radiations from the elements with atomic weights from that of tungsten to that of bismuth have not been examined minutely. They, however, appear to be very similar to the rays from Cu, Zn, &c. The ionization produced by these rays is of the order of magnitude of that produced by the rays from elements of the Cr–Zn group; but there appears slightly more variation in the character due to changes in the primary rays.

Though accurate observations have not been made, it appears probable that the radiation is a mixture of the scattered with the homogeneous rays, the proportional effect of the scattered being greater than in the radiation from Cu, Zn, &c.

Conclusions.

Secondary Röntgen rays of two distinct types are emitted by substances subject to a beam of X-rays. One, a scattered radiation produced by the motion of electrons controlled by the electric force in the primary Röntgen pulses, has been dealt with in previous papers by one of us, and has been further discussed in this paper with the H–S group of elements. The other, a homogeneous radiation characteristic of the element emitting it, and produced by the motion of electrons uncontrolled by the electric force in the primary pulses, has been but briefly mentioned.

All the phenomena of secondary X-rays so far observed by us may be explained by means of these two.

The experimental results of these investigations, both on the scattered radiation and the homogeneous radiation, are summarized below. In order to make the summary more complete, we have introduced several results which have been previously published. The references for these are given.

Experimental Results.—Scattered X-rays—those produced by the motion of electrons controlled by the primary pulses—constitute the bulk of the secondary radiation from elements of the H–S group when these are subject to a primary beam of low to moderate penetrating power.

Scattered X-rays are also emitted by many elements of

higher atomic weight—probably by all—when subject to such a primary beam; but unless the primary is very soft they are accompanied by homogeneous secondary X-rays (characteristic of the radiating element) which produce much greater ionizations.

The law of intensity of these scattered rays which holds for elements of the H–S group—that the intensity of radiation from an atom is proportional to its atomic weight—cannot be extended to include the elements of higher atomic weight. From some elements at least the intensity is greater than would be given by this law.

The scattered radiation from some elements whose characteristic homogeneous radiation is of comparatively penetrating type has been obtained free from admixture with this homogeneous radiation by the use of a primary beam consisting of less penetrating rays than the characteristic secondary.

Those scattered radiations not thus isolated have been accompanied by a homogeneous radiation more easily absorbed than the primary radiation producing them.

We have obtained no conclusive evidence that the relative intensity of secondary scattered and primary radiations changes with the penetrating power of the primary rays.

All elements of atomic weight greater than that of sulphur which have been examined emit a homogeneous secondary radiation when subject to a primary beam of X-rays of ordinary penetrating power. Cr, Fe, Co, Ni, Cu, Zn, Ag, have been examined. All other elements whose secondary radiations have been examined less minutely appear similar in this respect.

The penetrating power of this radiation from each element examined has been found independent of the intensity or the penetrating power of the primary radiation producing it; it is characteristic of the element emitting it.

The penetrating power of this radiation is a periodic function of the atomic weight of the radiating element (*e*).

The ionizing power of this radiation and almost certainly its energy is usually very much greater than that of the scattered radiation.

The homogeneous radiation has invariably been found more easily absorbed than the primary radiation producing it.

In all cases, when a primary was used which was softer than the characteristic homogeneous radiation, this radiation was not emitted. Also there is reason (from the curve connecting absorbability of a secondary radiation and the atomic weight of a radiator) for believing that those elements H–S which do not under ordinary circumstances emit such radiation, possess a characteristic radiation which is more penetrating than any primary beam used.

The intensity of this homogeneous radiation from copper is for a considerable range in the penetrating power of the primary merely proportional to the ionization produced by that primary in a thin film of air and is otherwise independent of the penetrating power of the primary. This is not general, as in many cases—probably all—the homogeneous radiation disappears when the primary radiation becomes more absorbable.

The intensity of the homogeneous rays in a given direction does not depend appreciably on the position of the plane of polarization of primary beam producing them.

The intensity of this radiation in a direction approximately opposite to that of propagation of the primary beam producing it, is within the small errors of experiment equal to that in a direction at right angles (*g*).

This radiation is specially penetrating to the element which emits it and to a less extent to elements of neighbouring atomic weight (*f*).

The fraction of the homogeneous rays from one element—copper—scattered by air, is the same as that for X-rays proceeding direct from an X-ray tube (within experimental errors) (*b*).

The absorbability of the secondary rays from copper which are scattered by air—tertiary rays—is the same as that for the direct secondary (*b*).

Theory.

The theory of the scattered X-rays has been dealt with in various papers, and the experimental evidence in support of that theory—briefly referred to in this paper—is so overwhelming that it need not be further discussed here.

It is important, however, to consider the evidence we have regarding the nature and origin of the homogeneous rays, which are characteristic of the elements emitting them.

The fact that the homogeneous secondary rays from copper are scattered by air in approximately the same proportion as the primary rays proceeding direct from an X-ray tube, and that the absorbability of these scattered rays is the same as that of the direct secondary rays, is strong evidence that they are of the nature of X-rays, for neither the observed intensity, nor the scattering without degradation, would have been expected on any corpuscular theory, whereas they are in perfect harmony with the æther pulse theory.

The relation between the absorption of a mixture of the homogeneous rays from a number of elements by various elements and the atomic weight of those absorbing elements is also very different from that found for any material radiation, while it is very similar to that obtained by experiments on a beam of Röntgen rays *.

The special powers of penetrating certain substances are such as have not been observed and are difficult to conceive of on any corpuscular theory.

Neither electrostatic nor magnetic deflexion of these rays has been observed.

Finally, the fact that the homogeneous rays are invariably produced by primary rays of more penetrating type, yet not necessarily more than just on the more penetrating side, appears some of the strongest evidence in favour of similarity in type between the primary and secondary rays. If the natures were different, the penetrating powers would represent totally different physical facts, and such connexion between them would be inconceivable. We can only conclude from consideration of this evidence that the nature of the homogeneous rays is similar to that of the primary X-rays.

As the homogeneous rays are of the nature of Röntgen rays, we must conclude that the radiation is set up by disturbance of electrons produced directly or indirectly by the passage of the primary pulses. That this motion of electrons

* Details of these experiments have not been given.

is not controlled by the electric forces in the primary pulses, is proved by the equality of the intensities of radiation in a direction approximately opposite to that of primary propagation and one at right angles, by the absence of evidence from the secondary rays of polarization of a primary beam in which such polarization exists, and by the absence of dependence of the penetrating power of the secondary beam on that of the primary.

The forces called into play which produce the accelerations resulting in radiation cannot then be directly due to the electric displacement in the primary pulses, but must be those called into play in the atom itself. The two possibilities that suggest themselves are that the radiations result from a disturbance of the atom, which quickly recovers its normal configuration, or that it is produced when the equilibrium of an atomic system is destroyed and forces of unusual magnitude are called into play.

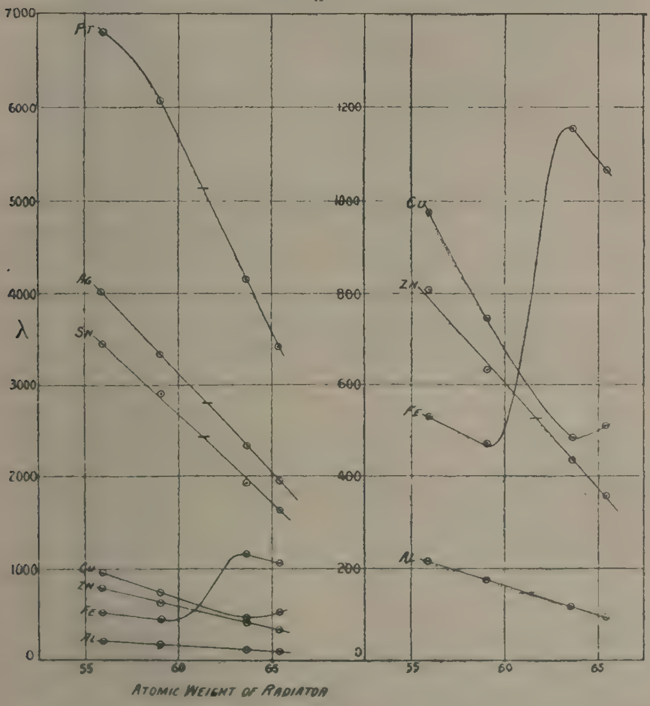
The homogeneity of the radiation and its independence of the primary rays suggest a regularity in the motion which is characteristic simply of the atom ; and although the relation between intensity of secondary radiation from copper and ionization produced by the primary in air is striking, it does not follow that the radiation is due to ionization in the radiating substance. Indeed, this appears highly improbaole, for the homogeneous radiation disappears when the primary radiation is made " soft " and appears in great intensity when the primary is "hard."

If, then, the radiation were emitted simply during the process of ionization and were proportional to it, a soft primary radiation would produce little or no ionization and a more penetrating radiation—(that is one more penetrating to most substances)—would produce an ionization at least hundreds of times as great. Though experiments have no been made on the ionizations produced in these substances investigated, such variations in ionization are of a higher order of magnitude than any observed. Again, ionization is undoubtedly produced in substances of the H–S group of elements when homogeneous rays are not emitted in appreciable intensity. We, however, do not know anything

of the homogeneous rays from these substances except that they are probably of penetrating type.

The relation between the absorption of a primary radiation and the intensity of secondary radiation emitted by the absorbing substance, also shows that the emission of homogeneous radiation necessitates a special absorption of the primary beam. Also, as far as we can estimate from experiments made, the energy of the homogeneous radiation is more than accounted for by the special absorption necessary to the production of that radiation.

Fig. 4.



Thus in fig. 4 we have plotted the coefficients of absorption of rays from Fe, Co, Ni, Cu, and Zn by Al, Zn, Ag, Sn, Pt In each case the radiations absorbed are not—if we generalize

from results given—capable of stimulating a homogeneous radiation characteristic of the absorbing substance, because these are of more penetrating type. The curves are similar. But when iron is used as the absorbing substance, cobalt radiation, which is just more penetrating, is unable to produce more than a feeble secondary in the iron, so the decline of the line for Fe is not quite so great as would have been expected, because of the little extra absorption necessary to set up this slight radiation from iron. The copper radiation, which is much more penetrating, is able to stimulate an intense radiation in iron and at the same time is enormously absorbed. This is exhibited by the sudden rise in the Fe curve. Beyond copper, the absorption appears to fall again according to the usual law, the absorption of the zinc radiation being a little less than that of the copper radiation. The stage is thus reached when the change in the penetrating power of the primary radiation produces no difference in the relative amounts of secondary homogeneous radiation and ionization by the primary beam, as shown by fig. 2 for the copper radiation. A study of the curves (fig. 3), exhibiting the transparency of various metals to homogeneous rays, shows the same thing. Thus, dealing with the radiation from zinc, aluminium, in which a Zn radiation does not stimulate a homogeneous secondary radiation is fairly transparent. Iron, in which an intense radiation is set up—the characteristic radiation from iron being considerably softer than that from zinc—is very opaque. Copper is much more transparent, as the zinc radiation being only a little more penetrating, is only able to set up a feeble radiation; while zinc, in which zinc radiation is unable to set up any further radiation, is more transparent still. The substances Ag, Sn, and Pt appear relatively more transparent to this radiation than to a more penetrating beam*. The zinc radiation is unable to set up radiations in these elements, as they are of more penetrating type. The absorptions are, however, not sufficiently abnormal to speak of with certainty. The effect is more clearly shown by the curve obtained from the homogeneous radiation from silver (fig. 3).

* Also Phil. Mag. Sept. 1907, p. 416, fig. 5.

These results show that a substance whose characteristic radiation is equally or more penetrating than the radiation incident upon it, does not absorb that radiation so much as when the incident is of more penetrating type and able to stimulate a secondary radiation in that substance. As the penetrating power of the incident radiation (as measured by most substances) increases, the absorption increases up to a certain point. A definite portion of the absorption thus appears to be connected with the secondary radiation, and may be proportional to it. But this does not conclusively show that ionization, or some kind of disruption in the atom is not the cause of the radiation, for the absorption of a certain amount of energy of the primary beam might be required to produce the instability which liberates more energy.

The facts that the homogeneous rays have invariably been found more easily absorbed than the primary rays producing them, and, in the cases investigated, that the homogeneous rays disappear when the primary becomes more easily absorbed, indicate a relation between primary and secondary which would be difficult to explain if this radiation were directly due to a disruption of any kind taking place in the atom. It appears rather that the radiation is due to what may be regarded as quite a normal behaviour of an atom after it has been passed over by Röntgen pulses, such as a free vibration of electrons.

Let us consider the passage of an electromagnetic pulse, in which the electric field is undirectional, over an electron with a free period of vibration much longer than double the time taken for the pulse to pass over it. (In this case the Röntgen pulse is thinner and more penetrating than the half-wave produced by the free vibration of the electron.) The electron receives an impulse and is left with kinetic energy after the pulse has passed. Consequently it is then acted upon by forces called into play in the atom itself. These produce a motion which is characteristic of the atom of which the electron forms a part, and this results in radiation—probably the homogeneous radiation discussed in this paper.

When the thickness of the primary pulse approaches the

half-wave length characteristic of the vibratory motion of the electron in the atom, the restoring force in the atom is brought into play before the primary pulse has passed and the absorption of energy and energy of subsequent radiation are diminished. This explains the diminished radiation when the penetrating power of the primary decreases and approximates to that of the radiation characteristic of the radiating substance. When the primary pulse is thicker than the half-wave characteristic of the motion of the electron, the electron is displaced a short distance and is gradually brought back by the restoring force against a gradually weakening electric force in the primary pulse, so that when this has passed, the electron is near its position of equilibrium again and the motion and radiation produced in the other cases are now absent. It is impossible to give an exact solution without some knowledge of the distribution of electric force in the primary pulses, of the forces binding the disturbed electron to the rest of the atomic system, and of the structure of that system; but we may consider this to be an approximation to the behaviour of each electron directly concerned with the phenomena discussed. The number of such electrons may not exceed one in each atom of radiating substance.

According to this theory, energy is taken from the primary beam and part, at least, appears as secondary homogeneous radiation, the rest being transformed into heat.

As the energy of this radiation is quite a considerable fraction of the total energy absorbed, we should expect that the difference between absorptions of primary rays more or less penetrating than the radiation characteristic of the absorbing element would be evident from a study of the absorption of the various homogeneous beams. But we have seen that there is a large absorption of a homogeneous radiation by an element which emits a much more easily absorbed radiation, because much of the energy is given to the electrons; that for other elements which emit a radiation only slightly softer, the absorption is much diminished because only a feeble disturbance and consequent secondary radiation is set up in the absorber; and when the radiation characteristic of the absorbing substance is more penetrating than

the absorbed radiation, the absorption is small and no homogeneous secondary radiation is produced. Thus what we have previously referred to as the special penetrating power of the homogeneous radiations may be explained by the small displacement produced in an atom by a radiation more easily absorbed, equally absorbed, or slightly more penetrating than the radiation characteristic of that atom, for reasons indicated.

Though according to such a theory, if the displaced electrons were merely held by a body of much greater mass, we should expect the emission of wave-trains instead of pulses, yet if the atomic system consisted of a number of interacting electrons, the energy of vibration would be rapidly communicated to other parts of the system and the motion of the displaced electron would be little more than half a complete vibration. The resultant radiation would in that case behave much as a number of isolated pulses.

An explanation on the disruption theory would be similar in many respects, but the displacement of electrons would on such a theory be sufficient to destroy the equilibrium of the atomic system and produce some change in its structure. The evidence against this is perhaps not conclusive, but there is no indication from the energy of secondary X-radiation of such a phenomenon, and the relation between the primary and secondary radiations points rather to the latter being due to the motion of the atomic system in regaining its normal configuration.

George Holt Physics Laboratory,
University of Liverpool,
30th May, 1908.

XXIV. *On the Coefficient of Diffusion.* By Basil W. Clack, *B.Sc., Lecturer in Physics at Birkbeck College*.*

[Plate XXIII.]

1. *Introduction.*

The following paper contains an account of a series of experiments on the diffusion of various salts through water. The objects with which they were undertaken were:—

1st. To test the practicability of a new method for the determination of the coefficient of diffusion;

2nd. To find how the value of the coefficient varies with the concentration of the solution.

Various methods have been employed from time to time to determine the value of the coefficient of diffusion. In some cases careful chemical analyses are necessary to obtain the amount of the substance diffused; in others it is the density of the solution which must be accurately determined at different points. Graham's, and kindred methods, involve serious disturbance of the solution in commencing an experiment, and also in withdrawing the various specimen layers for analysis.

The present investigation may be called a gravitational method, since the amount of the substance diffused is determined by means of the balance; but the method employed involves little disturbance at the commencement of an experiment, and during its progress the disturbance is quite negligible. Moreover, neither a series of density determi-

* Read May 22, 1908.

nations nor of chemical analyses is required to obtain the amount of diffusion which has taken place.

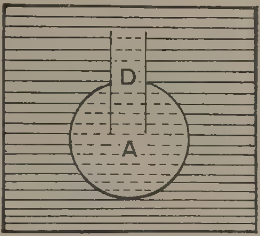
The method may be looked upon as an extension of that employed by Fick (Pogg. *Ann.* xciv. p. 59, 1855) and by Graham, but the method of obtaining the quantity of salt diffused is essentially different, as they determined this amount by chemical analysis.

Again, it does not appear that Fick took any account of the effect of the movement of the liquid along the tube, mentioned in the next section, on the quantity diffused. Although this omission would produce less effect in his method than in that about to be described, it would nevertheless cause a measurable difference in the calculated value of the coefficient of diffusion.

2. *Theory.*

In the diagram (fig. 1) let A represent a spherical vessel, to the upper side of which is fitted a vertical tube D of unit cross-section.

Fig. 1.



Suppose the bulb A and the tube D to be initially filled with a salt solution of uniform density, and to be suspended in a vessel containing pure water. Further, let it be assumed that the upper end of D is maintained in contact with pure water, and that the concentration of the salt solution at the lower end is also kept constant.

Under these conditions the salt will commence to diffuse up the tube D, and ultimately the diffusion current thus commenced will be found to become uniform.

Now when the steady state has been attained the quantities of water and of salt in tube D remain constant, and in dealing with variations in volume and weight we may confine our attention to the bulb A. When salt leaves A the volume of the liquid originally in it will decrease, since in general the addition of salt to water increases the resultant volume. Hence some of the water outside A will enter the apparatus

on account of this decrease in volume, with the result that as the salt diffuses up the tube D it meets a current of liquid which continually passes downwards, and which is due to the decrease in volume mentioned.

The weight of the suspended bulb A will decrease on account of the salt which leaves it, and will increase owing to the water which enters, and the resultant loss in weight is equal to the difference between the two.

Let d = density of solution at a point l cms. from top of tube D.
v = velocity downwards at the same point.
n = concentration in gms. per c.c. at the same point.
v_0 = velocity downwards at top of tube where $l=0$.
N = concentration in gms. per c.c. at the end of tube where $l=\mathrm{L}$.

With the degree of accuracy obtained in the present work, it is found sufficiently correct to assume $d=1+an$ where a is some constant.

Since the same mass of water crosses each section of the tube

$$v\,(d-n)=v_0,$$

i. e. $$v\,(1-bn)=v_0, \text{ where } b=1-a. \quad . \quad . \quad (\text{i.})$$

Now when the steady state is reached, the weight of salt which flows through each cross-section of the tube per second is constant $=c$, say

i. e. $-vn+k\dfrac{dn}{dl}=c$, a constant, where k = coeff. of diffusion;

or $$-\frac{v_0 n}{1-bn}+k\frac{dn}{dl}=c.$$

The solution of this equation is

$$-\frac{b\mathrm{N}}{v_0-bc}+\frac{v_0}{(v_0-bc)^2}\log_e\frac{(v_0-bc)\mathrm{N}+c}{c}=\frac{\mathrm{L}}{k}, \quad . \quad (\text{ii.})$$

which, as $(v_0-bc)\mathrm{N}$ is small compared with c, reduces to

$$\frac{\mathrm{N}}{c}\left(1-\frac{v_0\mathrm{N}}{2c}+\frac{v_0(v_0-bc)\mathrm{N}^2}{3c^2}\ldots\right)=\frac{\mathrm{L}}{k}. \quad . \quad . \quad (\text{iii.})$$

As already stated, there will be a contraction in volume produced in the vessel A when salt leaves it, and a corre-

sponding movement downwards will take place in the liquid in the tube D. The amount of water which enters the bulb owing to this movement may be calculated as follows:—

Mass of water in 1 c.c. of solution at bottom of tube D
= density of solution at bottom minus concentration of solution at bottom.
$=1+a\mathrm{N}-\mathrm{N}$,
$=1-(1-a)\mathrm{N}$,
$=1-b\mathrm{N}$.

Let $\delta=$ the ratio of the increment in volume produced to the increment in the mass of salt dissolved, for a solution of the strength employed in the bulb A; *i. e.* the increase in volume produced when 1 gram of salt is dissolved in a solution of the given concentration, the amount of the solution being so great that the addition of the salt makes no appreciable change in the concentration. The mass of water which enters the bulb due to a small diminution (dm) in the mass (m) of salt in the bulb

$$=\delta\times(1-b\mathrm{N})dm.$$

On integrating, the mass of water which enters the bulb due to a given diminution

$$=\int_{m_1}^{m_2}\delta\times(1-b\mathrm{N})dm.$$

Strictly both δ and N are functions of m, but the change in m is so small that they may both be taken as constants; thus the integral is equal to

$$\delta\times(1-b\mathrm{N})(m_2-m_1).$$

Now in one second $\qquad (m_2-m_1)=c.$

Hence the mass of water entering the bulb per second

$$=\delta\times(1-b\mathrm{N})c.$$

Now the decrease, i, in weight per second of the bulb = the weight of salt leaving per second minus the weight of water entering per second; or in symbols

$$i=c-c\delta(1-b\mathrm{N}),$$

$$\text{i. e.}\qquad c=\frac{i}{1-\delta(1-b\mathrm{N})}.$$

Again, the volume of water entering per second, $c\delta$, is equal to the velocity of the liquid at the bottom of the tube

$$=\frac{v_0}{1-b\mathrm{N}}, \quad . \quad . \quad . \quad . \quad \text{(from i.)}$$

$$\therefore \frac{v_0\mathrm{N}}{2c}=\frac{\mathrm{N}\delta}{2}(1-b\mathrm{N}).$$

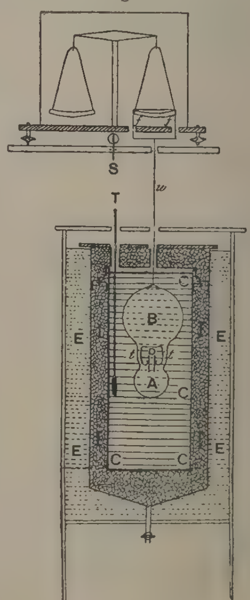
Again, the third term in the bracket in equation iii. is found to be negligibly small; hence this equation is written for practical use

$$k=\frac{\mathrm{L}i}{\mathrm{N}1-\left\{\frac{\mathrm{N}\delta}{2}(1-b\mathrm{N})\right\}\left\{1-\delta(1-b\mathrm{N})\right\}} \quad . \quad . \quad \text{(iv.)}$$

3. *Apparatus.*

A and B are two hollow spherical glass bulbs respectively 5 cms. and 9 cms. in diameter, connected together as shown by two parallel glass quill-tubes t, t (fig. 2).

Fig. 2.



On the upper side of A the bulb is pierced by an aperture into which the diffusion-tube D is hermetically sealed.

The ends of this tube were ground off perfectly plane and perpendicular to its axis, and its dimensions were, in most of the experiments, 4 cms. long, and 1·694 sq. cms. in cross-section.

The complete apparatus AB was filled with the solution under investigation, and suspended in a cylindrical copper vessel C of about 7 litres capacity, containing distilled water, the temperature of which may be kept constant by means of a surrounding ice-jacket I, protected by a layer of cork-dust E, several inches thick. The mercury thermometer T had a small range on each side of 0° C. and was divided into $\frac{1}{100}$° C.

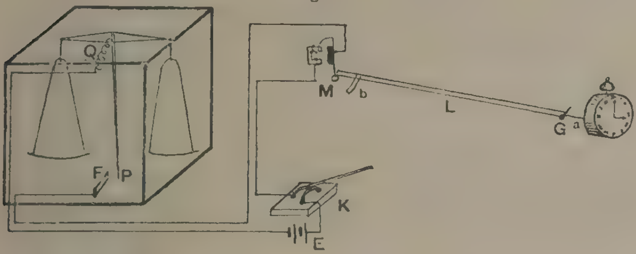
The Sartorius balance used was mounted upon a slate shelf firmly cemented into the stone wall of the laboratory.

Two holes were sand-blasted through the base of the balance-case, and a light frame, *f f*, passed through them as shown, and to this was attached a fine wire *w* which, passing through a third hole, in the slate shelf, sustained the apparatus already described. The movement of the balance beam was limited by means of stops shown at *s*, so that the pointer could only move over 1 scale-division, and the vertical movement which this permitted to the suspended system was only ·12 mm., so that any disturbance which might arise from this small and very slow movement will be quite negligible.

4. *Method of Experiment.*

As the salt diffuses up the tube, the stronger solution in the upper larger bulb, B, sinks by gravity, by means of the side tubes *t t*, and maintains the initial concentration of the solution at the lower end of the tube D; while the upper end is in contact with such a large volume of water that it may be considered to remain practically pure. The conditions studied in § 2 thus apply, and as diffusion proceeds the weight of the suspended system will decrease. If, then,

Fig. 3.



a slightly deficient weight be placed in the opposite scale-pan, the beam will turn until it reaches the supports. As time proceeds, however, the weight of the apparatus will ultimately become equal to that on the balance-pan, and the pointer will move to the zero of its scale. An automatic device was designed which would register the exact time at which this occurred. A very fine platinum wire P (fig. 3)

was soldered on to the lower end of the balance-pointer, and when the beam is in equilibrium, P makes contact with a fine strip of platinum-foil F, suitably mounted opposite the zero of the balance-scale. A delicate phosphor-bronze strip Q, such as is employed for galvanometer suspensions, served to connect the upper end of the pointer to a battery E, which was connected in series with a mercury lever-switch K, and an electromagnetic trigger M.

On contact being made between P and F the armature of the trigger is attracted, thus allowing a light wooden lath L, which is resting upon the trigger, to fall. This lath is pivoted at G, and is provided at this end with a flexible copper strip *a*, which, as the lath falls, is brought into contact with the balance-wheel of a common alarm-clock, from which the back has been removed, and which is suitably placed on the table.

The clock is thus stopped at the exact time at which the pointer reached the zero of the balance-scale.

In its fall another copper strip *b*, attached to the lath, knocks over the mercury switch K, and thus breaks the electric circuit.

The time recorded by the clock and the corresponding mass on the pan of the balance are recorded in tables, and from these the graphs following have been drawn.

A slightly smaller mass is then placed on the pan. The lath is replaced on the trigger, the clock reset and restarted, and the key K switched on, and the whole process repeated. This method, involving the determination of the time required for a given decrease in weight to take place, has an advantage over the converse process, in that it causes far less disturbance in the solution, and hence it has been adopted in the present investigation.

Another precaution which should be mentioned is that, the arrangement employed being extremely sensitive to changes in density, great care must be taken to prevent any but minute changes in temperature occurring in the solution.

The large amount of damping introduced, and the effect of the surface-tension of the liquid, prevented any accurate work being done until the effect of a film of oil on the surface of the solution was tested. This greatly reduces the surface-

tension, and increases the accuracy of weighing to a remarkable extent. Of a large number of oils tested, Fleuss pump-oil was found to be superior to any of the others, in that it always remained quite fluid.

As showing the effect of this oil-film a few figures might be quoted.

The apparatus was filled with water and suspended in water at a constant temperature. If set in oscillation, in no case would the balance-beam make more than two swings, and the resting place varied in different attempts between 10 and 15 on the scale for the same mass in the pan. Two drops of Fleuss pump-oil allowed to fall on the liquid near the suspending wire made a remarkable difference. Five turning-points could now be easily obtained, and assuming the ratio between any two consecutive deflexions to be constant, great consistency was found between the calculated positions of the resting place of the balance. For example,

Turning Points.		Resting Points.
20·8	3·2	10·0
14·4	7·8	10·2
11·5		10·1
		10·2
19·1	4·7	10·3
13·8	8·1	10·0
11·0		

Now in the diffusion experiments the beam does not oscillate, but slowly takes up its new position. The consistency obtained under these static conditions was tested, and the following is a sample of the figures obtained:—

Mass on pan in gms..........	37·105	37·104	37·103	37·105
Resting Point observed ...	10·2	11·9	13·5	10·0
Repeated.				
Resting Point observed ...	10·0	12·0	13·9	10·2

Moreover, it was found that the position of the pointer remained constant to within 1 scale-division for a week, the temperature being meanwhile kept at 0° C. by the surrounding ice-jacket.

The sensitiveness of the balance under the actual conditions holding in a diffusion experiment was found from a large number of readings to be ·00052 gm. per scale-division, *i. e.* a movement of 1 scale-division in the resting point was produced by ·00052 gm.

5. *Determination of the Concentration.*

Sodium chloride and potassium chloride prepared by Kahlbaum, of Berlin, were the first salts experimented upon. Solutions were prepared of approximately 0·2 gm., 0·1 gm., and 0·05 gm. of salt per c.c. of the solution, and in each case a check experiment was carried out to ensure experimental consistency. The concentration of these solutions was determined in most cases by three different methods: (1) From the density, making use of the results given in the *Physikalisch-Chemische Tabellen*, published by Landolt and Börnstein; (2) Precipitation by $AgNO_3$; (3) Evaporation to dryness in a platinum crucible.

The results obtained by all three methods usually agreed to within less than 1 per cent., as the following figures will show:—

Values of N in gms. of KCl per c.c. of Solution at 0° C.

Solution	2.	3.	4.	5.	6.	7.
By Density	·1990	·1009	·1004	·05122	·05147	·05106
By Precipitation	·1989	...	·1004	·05128	·05106	·05092
By Evaporation	·1988	·1008	·1004	·05100	·05097	·05113
Mean	·1989	·1008	·1004	·05117	·05117	·05104

The densities were obtained in the first place at the temperature of the laboratory; but the concentrations have been calculated at 0° C., the temperature of the experiments, by means of a determination of the absolute coefficient of expansion of the several solutions over this range of temperature, using for this purpose apparatus identical in principle with that employed by Dulong and Petit for a similar purpose.

6. *Determination of* δ.

It has been long known that when salts enter into solution, a change usually takes place in the volume occupied by the solvent.

F. Kohlrausch and Hallwachs (Wied. *Ann.* liii. 1894, p. 1) and Thomsen have studied these phenomena and have estimated the change in volume produced in several cases by the determination of the density of the solution formed, when a known mass of salt is dissolved in a known mass of water. W. F. Magie (Phys. Rev. xviii. 1904, pp. 449–452) and Macgregor have endeavoured to express in a formula the results obtained by previous experimenters.

We have seen that in the present investigation the phenomena referred to play an important part, and have deduced an expression for k involving the knowledge of the change in volume which accompanies a slight change in concentration.

Now δ is not a constant, but varies with the strength of the solution, and according to Magie depends on the relationship existing between the volumes occupied by the dissociated and the non-dissociated molecules of salt.

Its value can be easily deduced from the published tables giving the density of salt solutions of known concentration.

7. *Results for* KCl *and* KNO_3.

The accompanying graphs (Pl. XXIII.), which are numbered in accordance with the order of the experiments, show some of the curves obtained by plotting the weight of the suspended apparatus against the time as abscissæ.

The curves suggest that the method is at any rate capable of giving fairly consistent results, and they indicate a diminution in the coefficient of diffusion with a decrease in the concentration in the cases of both NaCl and KCl, a result which has been found by several other observers.

The actual figures obtained in this work are omitted for the reason mentioned below.

Before proceeding further, it was decided to test whether the outer vessel containing the salt solution was large enough. As the salt diffuses up the tube D the water outside becomes slightly denser, but the change in concentration is so small that it does not appreciably affect the quantity of salt diffused

up the tube; but the suspended apparatus has such a large volume, that a very slight variation in the density of the water outside has a considerable effect on the hydrostatic force upwards. When the earlier experiments were performed, the importance of this action was not fully appreciated.

It is impossible to calculate the change in the buoyancy of the water, but there is no doubt that the heavier liquid will tend to sink to the bottom, and that then diffusion will tend to make the density uniform. Any variation in the density of the liquid below the suspended apparatus will produce no effect on its weight, and so it is an advantage to have as deep a vessel as possible; to have the suspended apparatus as high as possible in the water; and to have the volume of the water as great as possible.

Experiments similar to the preceding were carried out with a deeper vessel, C C, shown in fig. 2, having also a volume about twice that of the original one, and they proved that the first vessel was undoubtedly too small. For this reason a number of results obtained with the earlier vessel must be discarded. The results given below have all been obtained with the larger vessel.

In the theory it has been assumed that the density of the solution at the bottom of the tube is constant over a horizontal plane, and that this density is the same as that of the mass of the solution in the neighbourhood. It has likewise been assumed that the density at the top of the tube in a horizontal plane is constant, and practically the same as that of pure water. These assumptions would only be strictly true if gravity were infinitely powerful, and in practice one would expect the density at the bottom of the tube to be rather less than that assumed, and at the top rather greater. These deviations from the assumptions will give a calculated value of k smaller than the true one, but even in the worse possible case studied, it can be shown by an approximate calculation that the error introduced is certainly much less than 1 per cent.

The narrower the tube the nearer will the assumptions be to the truth, but a narrow tube permits only a small amount of diffusion to take place.

In order to test whether it was necessary to have a tube of extremely narrow bore three diffusion-tubes were prepared, each 4 cms. long, but of different cross-section, as shown in the table below.

Using 10 per cent. KNO_3 solution, the following results were obtained using these tubes:—

Experiment.	$iA \times 10^6$ Mean Slope.	$k \times 10^5$.	Mean value $k \times 10^5$.	L.	A.
16	·2154	·843	·844	·00	1·694
17	·2159	·845			
18	·1294	·836	·843	3·990	1·024
19	·1316	·850			
20	·0956	·852	·853	3·993	0·743
21	·0959	·855			

The results obtained, using different tubes, do not vary more than the individual experiments on the same tube, so that for the present at any rate we may neglect any end correction to our tubes of 4 cms. length. The results also suggest that the vessel containing the water is large enough: for, although the quantity of salt diffused into the water varies considerably, the quantity in the last case being less than half that in the first, yet we get approximately the same calculated value for the coefficient of diffusion.

The next table gives the results for 5 per cent. solution of KNO_3.

L=4·00 cms.; A=1·694 sq. cms. N=gms. KNO_3 per c.c. solution at 0° C.

Experiment.	N.	$iA \times 10^6$.	$k \times 10^5$.	Mean $k \times 10^5$.
22	·0521	·1174	·870	·871
23	·0521	·1177	·872	

Comparing this result with the value obtained in Experiments 16 and 17 shown in the previous table, we see that the coefficient of diffusion of KNO_3 increases as the concentration decreases, as was noticed by Scheffer in 1888, a phenomenon contrary to that which holds for KCl and NaCl.

The results obtained with solutions of KCl are tabulated below.

L=4·00 cms. A=1·694 sq. cms. N=gms. KCl per c.c. solution at 0° C.

Experiment.	N.	$iA \times 10^6$.	$k \times 10^5$.	Mean $k \times 10^5$.	Approx. concentration.
24	·1975	·4748	·966	·972	20 p. cent.
25	·1975	·4810	·978		
26	·1034	·2518	·953	·954	10 p. cent.
27	·1021	·2491	·956		

8. *Earlier Method of Experiment.*

It might be mentioned that a number of experiments were carried out by a somewhat different method, denoted in the following section as Method A.

The diffusion apparatus, filled with distilled water, was inverted, so that the smaller bulb A (fig. 2) was uppermost, in a solution of the salt under investigation, thus allowing the salt to diffuse into instead of out of the apparatus, and the resultant rate of increase in weight was determined.

Some of the results obtained with solutions of KCl by this method are given in the next table in order to indicate the agreement.

L=4·00 cms. A=1·694 sq. cms. N=gms. KCl per c.c. solution at 0° C.

Experiment.	N.	$iA \times 10^6$.	$k \times 10^5$.	Mean $k \times 10^5$.	Approx. concentration.
8	·1997	·5729	1·011	1·009	20 p. cent.
9	·1997	·5703	1·007		
10	·1014	·2786	·952	·953	10 p. cent.
11	·1016	·2799	·955		

If the results of Experiments 26 and 27 be compared with those of 10 and 11, it will be seen that the agreement in the case of the 10 per cent. solution is almost perfect, but it is

not quite so satisfactory in the case of the 20 per cent. solution, for there is a difference of 3 per cent. The actual measurements are believed to be more accurate than this.

Some of the difference may be due to uncertainty in the value of δ, but this would not produce so great an error as 3 per cent. It may be that part of the difference is due to condensation of water from the air on the salt solution in Method A, so that part of the increase in weigh of the suspended apparatus may be due to the diminution of buoyancy of the solution, on account with its dilution with this condenced water.

The error produced by this cause would probably not be great, but no quantitative experiments were actually carried out on the subject, as the experience gained in the research convinces us that the first method described in this paper is distinctly superior to Method A.

For this reason no further experiments are being carried out by the earlier Method A.

Some of the points in which the method mentioned in § 4 shows superiority over Method A may be summarized. First, in this better method, the condensation of water mentioned above, instead of being harmful, is even somewhat of an advantage.

Again, this method necessitates the preparation of much smaller amounts of solution ; then it requires much less manipulation in commencing an experiment, as the apparatus, filled with the solution and having the open tube temporarily closed by a thin glass plate, may be simply lowered into the cooled distilled water.

Instead of this simple process, in the Method A in commencing an experiment, the solution under investigation was slowly run in through a glass tube passing to the bottom of the vessel C (fig. 2), while the distilled water, which is thus displaced, was removed from the top by means of a syphon. Great care was taken to ensure the complete removal of the water, so that the vessel C was filled with a solution of uniform density. . . . .

Thus the whole of the suspended system remains filled with distilled water, but is immersed in a salt solution of uniform density, so that the diffusion-tube, initially filled

with pure water, is brought into contact at its lower end with a salt solution whose concentration, on account of the large volume present, may be considered to remain constant.

Again, the better method enables an experiment to be repeated with precisely the same solution, so that the working details are much more under control. Moreover, it has another advantage in that greater accuracy is introduced in the determination of the quantity δ.

It is only in very dilute solutions that the value of δ is doubtful, and as only a very slight decrease occurs in the concentration of the solution inside the diffusion apparatus as the salt diffuses out, these very dilute solutions are not produced in this method.

Since no experiments were made by Kohlrausch with solutions as dilute as those produced by the diffusion of the salt into the suspended apparatus in Method A, it was necessary to make a special series of experiments to determine the value of δ. Moreover, as the solutions were so dilute, it was thought advisable to use a larger flask than than that employed by Kohlrausch. In place then of his flask of about 130 c.c. capacity, one was taken having a volume of about 300 c.c. and it was loaded with shot, so as to weigh about 4 gms. when immersed in water. The neck was drawn off and sealed up in the blowpipe, and the flask was then suspended by means of a pair of fibres of unspun silk attached to the pan of the balance, in distilled water cooled to 0° C.

It is very necessary that the fibres should be thoroughly wetted before use. They must also be smooth, and dust should be removed as far as possible from the water, otherwise the balance-beam refuses to swing, or only swings in a very erratic manner.

When the flask has been weighed in distilled water at 0° C. it is clamped by means of a suitable device, and a standardized solution of the salt is added to the distilled water (over 6 litres) 10 c.c. at a time. A series of very dilute solutions of known strength are thus obtained, and the corresponding density may be found, from which the value of δ easily follows.

NaCl.		KCl.		KNO_3.	
Concentration.	δ.	Concent.	δ.	Concent.	δ.
·0008453	·2152	·0004153	·3054	·0004146	·3097
·001265	·2208	·0008290	·3072	·0006211	·3072
1684	·2252	·001241	·3079	·0008267	·3061
2102	·2281	1652	·3074	·001032	·3042
2516	·2308	2061	·3089	1440	·3038
2931	·2329	2469	·3091	2047	·3037
3343	·2354	2875	·3094	2449	·3052
3754	·2363	3280	·3095	3046	·3061

It might be mentioned that a comparison of the weighings from which the above figures were obtained with the earlier ones mentioned in § 4, using a wire suspension, show that they are apparently equally consistent and trustworthy. It seems probable that the disadvantage of using such a delicate suspension as a fibre of unspun silk may be overcome by using a fine wire in its place, provided that the surface of the liquid be covered with a film of oil. When it is remembered that the fibre supports the flask inside a metal vessel surrounded by ice, and that clamping and stirring must be carried on without the experimenter being able to see what is happening inside the vessel, the advantage of using a somewhat stronger suspension will be apparent.

In the present experiments, however, the cocoon suspension was adhered to, and with this no oil-film should be used.

In order to find what value must be taken for δ in the various diffusion experiments, the method of successive approximations was used. The final rate of increase in weight is first assumed to be due entirely to the salt which enters the apparatus. In this way a rough value of the concentration inside the apparatus at the end of, say, a fortnight, is deduced. This gives us a rough value for δ, by means of which our first assumption may be corrected, and so yield a more exact value of δ.

9. *The Length of the Tube.*

Some experiments were also carried out with tubes of various lengths. It has already been shown that it is likely that to our present degree of accuracy no end correction is requisite to the tube of 4 cms. length. It may be pointed out that the end correction must depend on both the length and the diameter of the tube. The longer the tube the less is the rate of diffusion through it, and therefore the more perfectly can gravity regulate uniformity of density in a given horizontal plane.

Three tubes were employed, the lengths being 4 cms., 2 cms., and 1 cm.; the areas of cross-section being respectively 1·697, 1·025, and 1·059 sq. cms., and the diameters being 1·465, 1·14, and 1·16 cms. respectively; and the solution employed was a 5 per cent. solution of KCl. The results are plotted on the accompanying graph (Pl. XXIII. fig. 6).

Assuming no appreciable end correction to the tube of 4 cms. length, it is found that to give the same value for the Coefficient of Diffusion in each case, 0·036 cm. should be added to each end of the 2 cm. tube (*i. e.* ·032 of the diameter), and 0·045 cm. to each end of the 1 cm. tube (*i. e.* ·039 of the diameter). The results prove conclusively that it will not be possible to accelerate the work by using tubes much shorter than 4 cms., unless perhaps a battery of short and narrow tubes can be employed.

10. *Conclusion.*

My chief object in bringing this paper before the Physical Society of London in its present state has been the hope of obtaining from the Fellows some useful suggestions for future work, or improvements in the method.

In all 43 experiments have been performed; many of them, described in this paper, have had the sole object of endeavouring to obtain information of the best experimental conditions.

The numerical results of some of the others have not been stated, as improvements in the method, which they have suggested, have rendered the numerical result unsatisfactory.

The values of the coefficient of diffusion at 0° C., which the investigation indicates to be most trustworthy, are those obtained with the suspended apparatus filled with the salt solution, in the larger copper vessel containing distilled water. These results are

Experiment.	Salt KNO_3.		Experiment.	Salt KCl.	
	Concen.	$K \times 10^5$.		Concen.	$K \times 10^5$.
16	10 p. cent.	·843	24	20 p. cent.	·966
17	10 "	·845	25	20 "	·978
22	5 p. cent.	·870	26	10 p. cent.	·953
23	5 "	·871	27	10 "	·956

The results show that the method is capable of yielding consistent values, and if compared with those obtained by other observers in the same subject, although the concentration of the solutions and the temperatures at which they were employed vary considerably, thus making comparison difficult, yet the agreement, as far as can be judged, appears to be satisfactory.

For instance, results expressed in C.G.S. units, obtained by Schuhmeister and by Scheffer for KCl and for KNO_3 respectively are shown below

Salt.	Temp.	N.	$k \times 10^5$.	Observer.
KCl.........	17°·5	·022	1·47	Schuhmeister.
	10°	·007	1·27	
KNO_3......	7°	·031	·98	Scheffer.
	7°	·0093	1·06	
	10°	·015	·92	Schuhmeister.

The absence of any trustworthy values for the temperature

coefficients, and more especially the fact that the concentrations shown above are so much lower than those employed in the present paper, render further comparison difficult.

In conclusion, I should like to offer my thanks to the Principal and Council of Birkbeck College for assistance in carrying the experiments into effect, and especially to Dr. A. Griffiths, who not only suggested the research, but has been always ready to assist me when in trouble or difficulty.

Discussion.

Dr A. Griffiths congratulated the Author upon his paper, and referred to the importance of keeping the temperature constant during the experiments. The effects of temperature changes were very marked. The importance of a deep outer vessel was also surprising, and Dr Griffiths pointed out how it was possible to place a superior limit to the error introduced by the passage of the salt into the outer vessel. He also suggested the use of a differential method, using two glass vessels, to overcome some of the difficulties encountered in the experiments.

Prof. C. H. Lees said that all who had experience with diffusion experiments would congratulate the Author. The idea of using a balance was an old one, but the Author had made an old method a thoroughly reliable one. The question was still to be solved whether an increase of concentration increased or decreased the diffusion constant, and he hoped the Author would be able to carry on his experiments and settle this point.

XXV. *The Spectrum Top. By* F. Peake Sexton, *A.R.C.S.*, *Lecturer in Physics, Central Technical Schools, Truro* *.

[Abstract.]

The colours formed by the Benham spectrum top are here explained. The explanation rests on the fact that the red, green, and blue colour sensations have different rates of growth and decay.

When the top or disk shown in the figure is slowly rotated anticlockwise, it will be noticed that there is a slight red extension to the innermost sector lines. This indicates that the growth of the red sensation is the most rapid. If the rotation is reversed these lines present a blue colour, thus

* Read May 22, 1908.

showing that this is the last to decay. Thus there is good ground for assuming that the rates of growth are red, green, and blue, and of decay—blue, green, and red; where the first is the most rapid in both cases.

Experiments showed that the radial distance of the sector-lines was without effect on the colour of the lines, and the disk, as illustrated, shows that the colour depends on their angular positions.

These colour effects are dependent on the angular widths of the sector-lines, being best for lines of 1 mm. observed from 1 metre. This phenomenon is thus entirely dependent

on a contrast. With broad lines the colour produced is too faint to be observed, but when the line is narrow, the immediate surrounding white part will appear to have the complementary colour, and thus apparently increase the effect.

The length of the lines naturally affects the colour.

The speed of rotation has an important influence on the colour, but the best effects are obtained at about six revolutions per second.

When the illustrated disk is given an anticlockwise rotation, the colours are—purple—dull orange—yellow-green—violet-blue; counting from the centre, and this order of colours reverses with the rotation.

In order to obtain an explanation of the top, it is convenient to assume the eye as directed to one part and consider the succession of the different parts before it.

I. *The Innermost Sector-lines.*

The black sector will pass giving an impression of black lines on a red ground, but as there is not a contrast the effect is small. Then the sector-lines will pass giving a red impression behind them, because of the rapid growth of the red sensation, and this is reinforced by a contrast. Finally the disappearance of the white sector will give a bluish impression, which is not reinforced by contrast. Thus the resultant impression on the eye is *red lines.*

II. *The Outermost Sector-lines.*

The black sector disappears giving a red impression, which is unaided by a contrast. Then the sector-lines come into view with a blue after-image on them, which latter is reinforced by a contrast. Finally the black sector appears giving another blue impression, but in the absence of a contrast it is very slight. The resultant impression on the eye is *blue lines.*

III. *The Middle Sector-lines.*

The colours in these cases are simply due to a combination of the causes considered under I. and II. In all cases, the causes II. act first to produce a blue image, and those of I. second to produce a red image ; and these combine to give tne resultant colour. This depends on the relative intensity of the impressions, which vary with the size of the white sectors producing the effects. Thus the inner set of lines is orange because the red impression is about double that of the blue : and the outer set is green because the blue in this case is double the red.

The above experiments were all made in daylight, but Abney's experiments made under various illuminations fully confirm the theory which the author deduced.

The following references are to the past work on this subject :—

Prof. Liveing . . . . . Nature, vol. li. 1894–5.
Mr. C. E. Benham . . . " " "
Sir W. de W. Abney . . " " "
Mr. Shelford Bidwell . . Proc. Royal Society, 1896.

DISCUSSION.

Mr F. E. SMITH asked if the Author had viewed the top when illuminated with lights of different colours, and if so with what result.

The AUTHOR replied that experiments had not been tried in different coloured lights, as there was difficulty at the time of obtaining a coloured illumination of sufficient strength. The bands were visible in sodium light; but this did not affect the theory, because sodium light, although practically monochromatic, affects the three colour sensations.

XXVI. *A Modified Theory of Gravitation.* *By* C. V. BURTON, *D.Sc.**

1. So many questions arise in connexion with any theory which aims at assigning a dynamical basis for gravitation, that it may conduce to clearness if some elements of this paper are first presented without mathematical treatment.

The theory now put forward is pulsatory, and may in fact be regarded as a development of that which has been suggested by Prof. Hicks †. The æther, even where modified by the presence of atomic matter, is assumed to be so nearly incompressible that the bulk-modulus of elasticity enormously transcends every other elastic modulus; so that, in dealing with any compressional-rarefactional disturbances, we may treat the medium as a fluid.

In two particulars, however, there is an essential difference between the views now tentatively put forward and the more familiar form of pulsatory hypothesis. These differences are indicated in the following five paragraphs.

2. It is a well-known hydrodynamical result ‡ that if two spheres, placed at some distance apart in a frictionless incompressible fluid, are by any means caused to execute periodic pulsations (increase and decrease of volume), the pulsation-period being the same for the two spheres, the

* Read May 8, 1908.

† W. M. Hicks, "On the Problem of two Pulsating Spheres in a Fluid," Proc. Camb. Phil. Soc. iii. p. 277 (Oct. 1879).

‡ Hicks, *loc. cit.* *Cf.* also Basset's "Hydrodynamics," vol. i. chap. xi., where further references are given.

average force which either sphere causes to be exerted upon the other will be an attraction, a repulsion, or zero, according as the phase-difference of the pulsations is less than, greater than, or equal to a quarter-period. Hence, if we are to construct on these lines a model illustrative of gravitation, we must assume agreement of phase amongst all the pulsating centres, or at least we must suppose that the phases of the various centres are more nearly in agreement than would be the case if the phase-distribution were purely random. This seems to suggest that the source of pulsatory motion is not to be sought for in the free vibrations of individual atoms or electrons, but rather in something external to these, and acting on them all in common.

Let us suppose, for example, that free æther is very nearly incompressible, and that every region where atomic matter exists is a region of somewhat enhanced compressibility. On this supposition, an increase of pressure applied by any ideal means to all the æther within a given volume would cause only a very minute contraction except where atomic matter was present ; the expansion resulting from a diminution of the pressure applied to the æther being similarly localised.

3. It appears that we might thus account for attractions of gravitational type by supposing the ætherial pressure to be undergoing a *secular* change. A gradually increasing (or decreasing) pressure would cause every region containing atomic matter to behave as a sink (or as a source) in the medium. For our purpose, however, *accelerated* inflow (or outflow) of æther at each sink (or source) would be necessary ; that is to say, the strength of each sink (or source) must be steadily increasing or steadily decreasing with the time—no matter which. The suggested possibility of accounting for gravitation by means of a slow secular change taking place throughout the universe appeared at first sight very alluring ; but calculation shows at once that, feeble as gravitational forces are, they are incomparably greater than could be accounted for in this way.

4. What, then, would be the result of periodic or quasi-periodic fluctuations of the ætherial pressure? Every

portion of atomic matter would constitute a centre of pulsatory motion in the æther; and throughout any region not too immense the pulsations would agree sensibly in phase. It is by assuming the existence of a fluctuating ætherial pressure that I have here sought to obtain a consistent illustration of gravitational attraction; and, as furnishing most readily an assignable cause for such fluctuations of pressure, the propagation of compressional-rarefactional waves through the nearly incompressible æther is suggested. These waves are supposed to travel with so great a velocity that, even though all effective periods involved be very small, the effective wave-lengths, measured even by astronomical standards, are very great. Though the origin of the assumed wave-trains were unexplained, these might still be regarded as contributing to the explanation of gravity; for when once the constitution of a dynamical system has been defined, the mere supposition that the system is in motion is hardly to be viewed as a piling-up of hypotheses. And if we regard the question broadly, extending our consideration to that wider range of space and motion wherein the whole of our explorable universe must needs be treated as an infinitesimal volume-element, sensibly homogeneous as regards its ætherial content, we may admit our complete ignorance of this greater universe, and of the forms of disturbance which might emanate from it. One might perhaps conceive of compressional waves as proceeding from a more primitive and chaotic condition of the "primal æther," from which an æther such as we know, with electromagnetic qualities, may be in process of formation. As regards the sensible uniformity in time which must characterize the primary disturbance, if so unchanging a phenomenon as gravitation is thus to be accounted for, we may think of it as a quality naturally to be looked for in any activities whose scale is sufficiently vast; just as the energy of the radiation passing into space from the sun varies but little in the course of an hour, or as the turbulence of an ocean presents much the same aspect from minute to minute.

In any case it is instructive to trace out some of the consequences which might be expected to follow from the propagation of compressional waves through a very slightly

compressible æther; the more so as we are led to certain conclusions which, although essentially based on the ordinary postulates of dynamics, are at variance with some traditional or instinctive views.

5. The second innovation referred to concerns the nature and mode of progression of the centres of enhanced compressibility, which are assumed to be associated with atomic matter. If the nucleus of an electron is taken to be a vacuous cavity, the question of free mobility of this nucleus through the æther presents some difficulty, even when we suppose that the electron itself is essentially constituted by a self-equilibrating distribution of strain in the æther surrounding the nucleus. But now, discarding for the moment a too minute scrutiny of ætherial constitution, so that we may regard the æther as a continuous medium, let us suppose that the nucleus of an electron *, instead of being vacuous, is merely a region of somewhat diminished density. (Though increased compressibility may not be inevitably associated with diminished density, it is convenient to assume such a relation for preliminary descriptive purposes. In the sequel this question is treated more generally.) This implies that, in the complete strain-distribution which constitutes the electron, there is included some degree of expansion in the nuclear region. An electron thus constituted may more readily be conceived as freely mobile through the æther, no vacuous cavity being present to complicate the problem; so that any infinitesimal displacement of the electron is simply equivalent to the impressing of a differential strain upon the medium.

6. This assumption as to the nature of an electronic nucleus is admittedly gratuitous, but apart from the difficulty regarding mobility which it was designed to remove, it has the advantage of greatly simplifying the dynamics of the problem proposed. So far as we are concerned with the distribution of resultant ætherial motion from point to point, we may treat each moving electron as a doublet comprising a source and a neighbouring sink, while it is shown that the

* The necessity for distinguishing, in this connexion, between positive and negative electrons is considered in Appendices A and B.

resultant of the hydrodynamical (gravitational) forces * tending to accelerate the motion of the electron simply follows the gradient of diminishing pressure and is proportional thereto. It further appears that the assumption referred to might enable the elementary requirements of the gravitation problem to be satisfied with the contribution of only a very minute non-electromagnetic term to the total effective inertia of an electron (Appendix D).

7. Let F be a physical magnitude characteristic of all electrically neutral (atomic) matter, and defined as follows: if m is the mass of atomic matter included in any given region, then $\mathrm{F}m\rho$ is the defect of mass of ætherial substance contained within that region, as compared with the mass included in an equal region filled with free æther; the density of the æther being denoted by ρ. (As we shall suppose the average density of ætherial substance to be but little different in a vacuum and in the densest atomic matter, no more precise definition of ρ is needed in this connexion.) Otherwise thus: if, within a region bounded by a fixed ideal surface and originally free from atomic matter, a mass m of matter could be created, a volume $\mathrm{F}m$ of æther would flow outward across the bounding surface. F may be called the extrusion of æther per unit mass of matter.

8. Consider further the application of a small additional normal pressure δp to a surface bounding some definite portion of the ætherial medium, which is modified by the presence of a mass m of atomic matter. A reduction of volume will result, and we shall define another constant H by the statement that the reduction of volume is *greater* by $\mathrm{H}m\delta p$ than if the original volume had been wholly occupied by free æther. Thus evidently

$$\mathrm{H} = -d\mathrm{F}/dp. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

9. Let a particle of atomic matter be present where from any cause there is a pressure-gradient in the æther. When

* That is to say of those forces which are concerned in palpable ætherial motions; our scrutiny being minute enough to take account of variations of ætherial qualities through the region occupied by an electron, but not so minute that the æther ceases to appear to us as a continuous medium.

the particle is displaced from one point to a second neighbouring point, there is a flow of æther inward at the first point, and a flow outward at the second point. If the ætherial normal pressure is greater at the first point than at the second, work on the whole will be done by the pressure during the displacement.

Consider a particle m of matter at a point (x, y, z) where the pressure is p, and let the particle be displaced to $(x+\delta x, y, z)$ where the pressure is $p+\partial p/\partial x \, . \, \delta x$. Also let the particle be acted upon by such ideal forcive as may be necessary to ensure that the displacement takes place very slowly. There is work done by the pressure p at (x, y, z) corresponding to a reduction of volume $\mathrm{F}m$; this work being $p\mathrm{F}m$. There is also work done against the pressure $p+\delta p$ at $(x+\delta x, y, z)$ corresponding to an increment of volume $(\mathrm{F}+\delta\mathrm{F})m$; this work is, to a first order,

$$(p\mathrm{F}+p\delta\mathrm{F}+\mathrm{F}\delta p)m,$$

where $$\delta p = \partial p/\partial x \, . \, \delta x,$$

and $$\delta\mathrm{F} = -\mathrm{H}\delta p = -\mathrm{H}\partial p/\partial x \, . \, \delta x.$$

On the whole, then, the work done by the normal pressure of the medium in the virtual displacement δx is

$$\left(\mathrm{H}\frac{\partial p}{\partial x} \, . \, p - \mathrm{F}\frac{\partial p}{\partial x}\right)m\delta x.$$

Of this the first term represents the work done in compression, the potential energy of the system being increased to that extent, while the remaining term is the work done against the ideal forcive which was applied to the particle to prevent its moving with appreciable acceleration. Thus if a particle of mass m is at rest in the æther (or, as below, in motion) at a point where an ætherial pressure-gradient exists, there will be a force

$$-m\mathrm{F}\left(\frac{\partial p}{\partial x}, \ \frac{\partial p}{\partial y}, \ \frac{\partial p}{\partial z}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

exerted on the particle. If the particle were allowed to move with a finite acceleration, the associated movements of expansion and contraction would be correspondingly accelerated, and work would thus be done in imparting this

form of kinetic energy to the æther. This only implies, however, the existence of a corresponding term * in the total effective inertia of the particle, the remaining terms being probably of electromagnetic origin. Since here, as in the usual acceptation of the term, the mass m signifies the *total* inertia of the particle in question, we may remove the restriction as to the particle being at rest, (2) being in any case the force-components acting on m. It should be particularly noticed that the virtual displacement from which (2) was deduced was a displacement of m with respect to the æther, and that consequently

$$-m\mathrm{F}\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right) \textit{ is the force tending to}$$
accelerate the particle m *with respect to the æther* . (3)

It will be necessary later to examine the forces which material bodies might be expected to exert on one another owing to their motion through the æther.

10. Properly speaking, there are two distinct ways in which matter may be conceived of as moving with respect to the æther. For simplicity, consider only a single electron, and let us agree to give the name "nucleus" to a certain definite central region of the electron. At any instant the nucleus comprises a certain identical portion of ætherial substance, and if this portion were bodily displaced, with respect to the surrounding æther (which thereby became strained), it might be said that the electron had suffered a displacement of like amount. But such displacement, even through distances far less than the diameter of the electronic nucleus, would presumably be met by an opposing stress of enormous magnitude, arising from the deformation of the circumnuclear æther; while the alternative type of electronic motion, involving only transference of strain from one part of the æther to another, can take place freely without evoking any opposing stress. (The forces mutually exerted by electrons and assemblages of electrons not coming here into consideration.) We shall suppose that when, from any cause, matter experiences a tendency to move, or to change

* The possible magnitude of this term is considered in Appendix D.

its motion with respect to the æther at large, the resulting motion may, with sufficient accuracy, be treated as exclusively of the strain-transference type assumed in the last paragraph. It should be remarked that, whereas vortex motion would be involved in a bodily displacement of that identical portion of ætherial substance which is instantaneously co-extensive with the nucleus of an electron, the type of motion dealt with in § 9 above is irrotational. The latter motion may be supposed to be entirely distinct from that which, arising from the charge of the moving electron, constitutes a surrounding magnetic field. Such a supposition is justifiable if, as I imagine, a magnetic field involves no bodily motion of the æther.

11. Let us now conceive the æther to be traversed by trains of compressional-rarefactional waves, and consider first how the motion of matter will be *directly* affected thereby. As already stated, the æther is regarded as so slightly compressible that, where compressional waves are concerned, any stresses which may be involved, over and above the variations of normal pressure, are relatively insignificant: the æther, in regard to such waves, behaving like a compressible fluid, whose motion is sensibly irrotational. To fix ideas, let the wave-motion be limited to a single harmonic train of plane-waves, propagated with velocity $\mathfrak{V}$ in the direction of x-increasing. It will be convenient to define the waves in terms of the variations of pressure taking place in the æther; thus

$$p = \bar{p} + \mathrm{B} \sin \frac{2\pi}{\lambda}(x - \mathfrak{V}t + \epsilon); \quad . \quad . \quad . \quad (4)$$

so that B is the semi-amplitude of pressure-variation, $\mathfrak{V}$ the velocity of the disturbance, λ the wave-length, and $\lambda/\mathfrak{V}$ the period; while ϵ is a linear magnitude serving to fix the otherwise undefined question of phase.

12. As a particular case, suppose that F, the extrusion of æther per unit-mass of atomic matter (§ 7), is zero, any given volume of space thus containing precisely the same amount of ætherial substance, whether atomic matter is present in that space or not. It is then evident from general considerations that the æther affected by the wave-motion, in

surging to and fro, will merely carry with it whatever atomic matter may be present; no tendency arising for those strain-distributions which constitute matter to lag behind or to outrun the bodily excursions of the ætherial plenum. The same conclusion is reached when we put $F = 0$ in (3); there are then no forces belonging to the class there concerned, or, in other words, the wave-motion has no tendency to accelerate the motion of atomic matter with respect to the æther; so that every portion of atomic matter will continue in its state of rest, or of uniform or accelerated motion *with respect to the æther*, precisely as if no wave-motion were taking place. Under this particular assumption that $F = 0$, the wave-motion (at least in the case of sufficiently great wave-lengths) would appear to be without influence on any of the phenomena which come within the range of our observation, and to belong to that class of activities for which the name *aphenomenal* has been suggested *. Apart from any question as to the correspondence between our special assumptions and the conditions obtaining in the physical universe, it is instructive to realize that the picture here presented—of vibratory motions affecting all matter without influencing the phenomena of ordinary dynamics—is in no way incompatible with the fundamental concepts of dynamical science.

13. If, instead of $F=0$, as in § 12, we suppose F to be finite, so that the mean density of ætherial substance in a region containing atomic matter is different from the density in free æther, it is evident that a general bodily acceleration of the æther will tend to produce an acceleration of atomic matter with respect to the æther. From (3) and (4) we find for the acceleration of any free mass m of neutral matter with respect to the æther

$$a = \left\{ -\mathrm{FB}\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(x - \mathfrak{V}t + \epsilon),\, 0,\, 0 \right\}; \quad . \quad (5)$$

while the absolute acceleration A of the æther itself is given by

$$\mathrm{A} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right) = \frac{1}{\rho}\left\{ -\mathrm{B}\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(x - \mathfrak{V}t + \epsilon), 0, 0 \right\}. \quad (6)$$

* But see also Appendix B.

In this case then, and (as we easily see) generally for small compressional disturbances of the æther, the ratio

$$\textit{acceleration of free matter with respect to the æther} : \textit{acceleration of the æther} = \mathrm{F}\rho \quad . \quad . \quad . \quad . \quad . \quad (7)$$

14. There is another aspect of the hypothetical wave-motion which deserves some attention. At any point fixed with respect to the æther, changes of pressure are supposed to occur, accompanied by changes of density, and as we consider the compressibility of the medium in bulk to be very slight, the proportional changes of pressure and of constitutive (potential) energy will be great compared with the proportional changes of density. From this it may be surmised that corresponding changes in the velocity of radiation would be involved, and if the amplitude of these changes were not a quite insignificant fraction of the whole velocity, effects might result which would be palpable to observation. On the other hand, all effective wave-lengths are assumed to be so immense that, throughout even an astronomically considerable region, the ætherial pressure, and therewith the velocity of radiation, may be treated as uniform at each instant. It is perhaps conceivable that the effect of increased ætherial pressure would be to accelerate not only radiation but *all* phenomena in the same proportion; so that, for example, two universes identically constituted otherwise, but differing from one another as to the pressure prevailing throughout the æther and the rapidity of phenomena in general, might present identical aspects to their respective inhabitants. But even so, in the question whether the pressure of the æther throughout our universe could be gradually or suddenly changed without giving rise to observable effects, some nice points of dynamics would seem to be involved *.

15. The primary effects of the hypothetical compressional waves having been for the moment disposed of, the secondary

* Among the values which, in § 33 below, are tentatively assigned to the various physical magnitudes involved, the variations of potential energy of the æther are set down as so small in comparison with the whole constitutive energy that the special difficulty above referred to seems to be evaded.

or gravitational effects may be considered. Every portion of atomic matter, in so far as its presence implies altered compressibility of the ætherial plenum ($H \neq 0$), behaves as a centre of pulsatory motion, or as an aggregation of such centres. Thus the motion of the æther, to a second approximation, is to be found by superposing on the wave-motion proper a pulsatory motion due to a distribution of alternating source-sink centres, to which should be added (except, as will appear, when $F=0$) a distribution of source-sink doublets arising from the *motion* of atomic matter through the æther. It will be assumed that the whole region of space under consideration is so circumscribed that its greatest dimension is very small compared with the shortest effective wave-length of the compressional disturbance, so that, at any given instant, the primary pressure-variation is sensibly in the same phase throughout, while, to the same order of approximation, the source-sink centres just referred to may be treated as if they existed in an incompressible medium. In accordance with the assumptions explained in §§ 1, 11 above, the variations of normal (hydrostatic) pressure are taken to outweigh in importance all other stresses evoked by varying compression of the medium, and the whole motion to be investigated is accordingly irrotational.

16. Let σ be the density of atomic matter at any point (x, y, z); then the total compressibility of matter-encumbered æther contained in the volume-element $dx\,dy\,dz$ exceeds the total compressibility of a like volume of free æther by

$$H\sigma dx\,dy\,dz, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in accordance with our definition of H (§ 8). Further, so far as the primary disturbance is concerned, let the pressure p be expressed by

$$p = \bar{p} + \mathfrak{p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

the presence of atomic matter in the volume $dx\,dy\,dz$ is thus equivalent to a *source* of strength

$$-H\frac{\partial \mathfrak{p}}{\partial t}\sigma dx\,dy\,dz; \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which expression must therefore be equated to $\nabla^2\phi\,dx\,dy\,dz$, ϕ being the velocity-potential of the secondary motion now considered, while $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$.

With due regard to boundary conditions, this leads to

$$\phi = \frac{H}{4\pi}\frac{\partial \mathfrak{p}}{\partial t}\iiint \frac{\sigma'}{r}\,dx'dy'dz' \; ; \quad . \quad . \quad . \quad (11)$$

σ' being the density of atomic matter at any point (x', y', z'), while $r^2 \equiv (x-x')^2+(y-y')^2+(z-z')^2$. The integration must be conducted through a sufficiently extended region to include all pulsatory centres which contribute appreciably to the value of ϕ at (x, y, z). In accordance with the assumption mentioned in § 15 above, the factor $\partial\mathfrak{p}/\partial t$ is placed outside the sign of integration; while sources and sinks other than those corresponding to the expression (10) are assumed to be absent from the secondary motion.

17. Treating the primary wave-motion more generally than hitherto, (4) may be taken as expressing a single typical constituent, no restriction being imposed on the direction of propagation of the waves, or on their periods or phases, provided that all effective wave-lengths are supposed sufficiently great, and all limited sources of the primary disturbance sufficiently remote. The form of our equations of course involves the implicit assumption that, in the primary wave-motion, all relations are linear. It is now similarly assumed that, in dealing with the secondary disturbance expressed by (11), non-linear terms may be omitted from the equations, the principle of superposition being accordingly applicable to the primary and secondary motions. Thus, to our order of approximation, we may write

$$p = \bar{p} + \mathfrak{p} + \varpi \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and

$$\frac{\partial\psi}{\partial t} + \frac{\partial\phi}{\partial t} = -\int\frac{dp}{\rho} + \text{func.}\,(t)$$

$$= -\int\frac{d(\mathfrak{p}+\varpi)}{\rho} + \text{func.}\,(t)\; ; \quad . \quad . \quad (13)$$

where ϕ, ϖ are respectively the velocity-potential and pressure-increment corresponding to the secondary motion, ψ, $\mathfrak{p}$ being the velocity-potential and pressure-increment corresponding to the primary disturbance. Hence

$$\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial t} + \frac{\partial\phi}{\partial t}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{1}{\rho}\left(\frac{\partial\mathfrak{p}}{\partial x} + \frac{\partial\varpi}{\partial x}\right); \quad . \quad (14)$$

and subtracting from this the equation proper to the primary motion, there remains

$$\frac{\partial^2\phi}{\partial t\,.\,\partial x}=-\frac{1}{\rho}\frac{\partial\varpi}{\partial x}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

which with (11) gives

$$\frac{\partial\varpi}{\partial x}=-\frac{\mathrm{H}\rho}{4\pi}\cdot\frac{\partial^2\mathfrak{p}}{\partial t^2}\iiint\sigma'\frac{\partial}{\partial x}\left(\frac{1}{r}\right)dx'\,dy'\,dz'$$

$$=\frac{\mathrm{H}\rho}{4\pi}\frac{\partial^2\mathfrak{p}}{\partial t^2}\mathrm{I}_x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where

$$\mathrm{I}_x=\iiint\frac{\sigma'}{r^2}\,\frac{x-x'}{r}dx'\,dy'\,dz'. \quad . \quad . \quad . \quad (17)$$

Hence the secondary motion gives rise to a force upon a mass m of atomic matter at (x, y, z), the x-component of which is, by (2),

$$-m\mathrm{F}\frac{\partial\varpi}{\partial x}=-m\frac{\mathrm{HF}\rho}{4\pi}\frac{\partial^2\mathfrak{p}}{\partial t^2}\mathrm{I}_x=m\mathrm{X}\text{ (say)}, \quad . \quad (18)$$

with corresponding expressions for the y- and z-components.

In (18) F may be written $\overline{\mathrm{F}}+\delta\mathrm{F}$, where $\overline{\mathrm{F}}$ is the mean value of F, and $\delta\mathrm{F}$, to a sufficient approximation, represents a variable term, arising from and proportional to the variable pressure-term $\mathfrak{p}$, which corresponds to the primary disturbance. Thus

$$\mathrm{F}=\overline{\mathrm{F}}+\delta\mathrm{F}=\overline{\mathrm{F}}-\mathrm{H}\mathfrak{p}, \quad . \quad . \quad . \quad . \quad (19)$$

by (1); so that (18) becomes

$$\mathrm{X}=\frac{\rho}{4\pi}\left(-\mathrm{H}\overline{\mathrm{F}}\frac{\partial^2\mathfrak{p}}{\partial t^2}+\mathrm{H}^2\mathfrak{p}\frac{\partial^2\mathfrak{p}}{\partial t^2}\right)\mathrm{I}_x. \quad . \quad . \quad (20)$$

The mean value of X is therefore given by

$$\overline{\mathrm{X}}=\text{av.}\left[\frac{\rho}{4\pi}\mathrm{H}^2\mathfrak{p}\frac{\partial^2\mathfrak{p}}{\partial t^2}\mathrm{I}_x\right]. \quad . \quad . \quad . \quad (21)$$

18. As the primary wave-motion is now supposed to involve only very great wave-lengths, so that the (primary) pressure-variation throughout the region of space considered is sensibly a function of the time only, and not of x, y, z, we may write

$$\mathfrak{p}=\Sigma\mathrm{B}_s\sin(st+\epsilon_s); \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

each B being of the nature of a pressure (in general infinitesimal), while the ϵ's are phase-terms. Hence

$$\frac{\partial^2 \mathfrak{p}}{\partial t^2} = -\Sigma \mathrm{B}_s s^2 \sin(st+\epsilon_s); \quad . \quad . \quad . \quad . \quad (23)$$

and (20) becomes

$$\mathrm{X} = \frac{\rho}{4\pi} \mathrm{H}\overline{\mathrm{F}}\Sigma \mathrm{B}_s s^2 \sin(st+\epsilon_s) \,.\, \mathrm{I}_x$$

$$-\frac{\rho}{4\pi} \mathrm{H}^2 \Sigma \mathrm{B}_s \sin(st+\epsilon_s) . \Sigma \mathrm{B}_{s'} s^2 \sin(st+\epsilon_s) \,.\, \mathrm{I}_x. \qquad (24)$$

There is thus a gravitative field at (x, y, z) defined by the average values

$$(\overline{\mathrm{X}}, \overline{\mathrm{Y}}, \overline{\mathrm{Z}}) = -\frac{\mathrm{H}^2\rho}{8\pi}\Sigma \mathrm{B}_s^2 s^2 (\mathrm{I}_x, \mathrm{I}_y, \mathrm{I}_z)$$

$$= -\frac{\mathrm{H}^2\rho}{8\pi}\Sigma \mathrm{B}_s^2 s^2 \iiint \frac{\sigma'}{r^2} \frac{x-x', y-y', z-z'}{r} \, dx'dy'dz' \quad . \quad . \quad . \quad (25)$$

Within the limits of our assumptions, there is seen to be universal mutual attraction of electrically neutral matter, the Newtonian constant being

$$\mathrm{G} = \frac{\mathrm{H}^2\rho}{8\pi}\Sigma \mathrm{B}_s^2 s^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

19. It has been pointed out above, as an assumption essential to the theory proposed, that every effective wave-length of the primary ætherial disturbance must be very great compared with the distance between any mutually attractive bodies for which the Newtonian law of inverse squares has been closely verified. It may be worth while to indicate very briefly what would result if this condition were not realized. By way of illustration, let the primary disturbance take the form of plane progressive waves of harmonic type and of definite wave-length λ. Two bodies whose line of centres was perpendicular to the direction of propagation of the waves, and which were at a distance r apart, would attract one another with a force proportional to

$$r^{-2} \cos 2\pi r/\lambda,$$

which, as r is increased, changes sign periodically at intervals of $\frac{1}{2}\lambda$. (If, in place of a single wave-length λ, there were a continuous distribution of wave-energy over a wide range of wave-lengths, there would simply be a falling off of attraction

between the two bodies at a rate more rapid than that of the inverse square of the distance.)

With the primary disturbance in the form of progressive waves travelling in one direction (as above) the case of two bodies in a line not perpendicular to that direction would be more involved; but if the primary waves were travelling indifferently in all directions, there would merely be a gradual extinction of gravitational attraction as the bodies concerned were removed to distances apart not wholly inconsiderable in comparison with the smallest effective wave-length of the primary disturbance.

It will be remarked that, when the distance between two bodies is small compared with the mean effective wave-length of the primary disturbance, the attraction between them exhibits only a *second order* deviation from the Newtonian law of gravitation.

Motional Forces.

20. The next question to be considered concerns the forces arising from the motion of neutral matter with respect to the æther. The expressions for these forces, as will appear, contain a factor F^2, F being (as defined in § 7) the extrusion of æther per unit mass of matter: so that if the mean value $\bar{F}$ were zero, these forces—which correspond to nothing so far observed—would disappear.

Let (x, y, z) be the co-ordinates of a particle of mass m at time t, and let (x', y', z') be the simultaneous co-ordinates of another particle of mass m', all referred to axes fixed with respect to the general mass of æther. As in § 10, the type of ætherial motion which here concerns us, and which arises from the motion of m and m', with respect to the æther, is sensibly irrotational.

Let us consider the forces exerted on m owing to the motion of m', and let ϕ be the velocity-potential at (x, y, z) due to the motion of m' only, while $r^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$. Then, the effect of the motion of m' being representable by a source-sink doublet at (x', y', z') we readily find

$$\phi = -\frac{Fm'}{4\pi} \cdot \frac{d'}{dt}\left(\frac{1}{r}\right), \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where d'/dt operating on any quantity represents the time-flux of that quantity due to variations of x', y', z' only. Hence also,

$$
\begin{aligned}
\dot{\phi} &= -\frac{\mathrm{F}m'}{4\pi}\frac{d'^2}{dt^2}\left(\frac{1}{r}\right) \\
&= \frac{\mathrm{F}m'}{4\pi}\left\{\left(\frac{1}{r^3}-\frac{3(x-x')^2}{r^5}\right)\left(\frac{dx'}{dt}\right)^2+\ldots+\ldots\right. \\
&\quad -\frac{6(y-y')(z-z')}{r^5}\frac{dx'}{dt}\cdot\frac{dy'}{dt}-\ldots-\ldots \\
&\quad \left.-\frac{x-x'}{r^3}\frac{d^2x'}{dt^2}-\frac{y-y'}{r^3}\frac{d^2y'}{dt^2}-\frac{z-z'}{r^3}\frac{d^2z'}{dt^2}\right\}. \quad . \quad . \quad (28)
\end{aligned}
$$

21. In accordance with (3), the force components on the particle m at (x, y, z) are

$$-m\mathrm{F}\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right)=m(\mathrm{X}, \mathrm{Y}, \mathrm{Z}), \text{ say};$$

or, to a first approximation, neglecting squares and products of ætherial velocities,

$$
\begin{aligned}
m(\mathrm{X}, \mathrm{Y}, \mathrm{Z}) &= -m\mathrm{F}\rho\left(\frac{\partial\dot{\phi}}{\partial x}, \frac{\partial\dot{\phi}}{\partial y}, \frac{\partial\dot{\phi}}{\partial z}\right) \\
&= -\frac{mm'\mathrm{F}^2\rho}{4\pi}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\frac{d'^2}{dt^2}\left(\frac{1}{r}\right);
\end{aligned}
$$

that is

$$
\begin{aligned}
m\mathrm{X} = -\frac{mm'\mathrm{F}^2\rho}{4\pi}&\left\{\left(\frac{9(x-x')}{r^5}-\frac{15(x-x')^3}{r^7}\right)\left(\frac{dx'}{dt}\right)^2\right. \\
&+\left(\frac{3(x-x')}{r^5}-\frac{15(y-y')^2(x-x')}{r^7}\right)\left(\frac{dy'}{dt}\right)^2+\left(\frac{3(x-x')}{r^5}-\frac{15(z-z')^2(x-x')}{r^7}\right)\left(\frac{dz'}{dt}\right)^2 \\
&+\left(\frac{6(y-y')}{r^5}-\frac{30(x-x')^2(y-y')}{r^7}\right)\frac{dx'}{dt}\frac{dy'}{dt}+\left(\frac{6(z-z')}{r^5}-\frac{30(x-x')^2(z-z')}{r^7}\right)\frac{dx'}{dt}\frac{dz'}{dt} \\
&-\frac{30(x-x')(y-y')(z-z')}{r^7}\frac{dy'}{dt}\frac{dz'}{dt}+\left(\frac{1}{r^3}-\frac{3(x-x')^2}{r^5}\right)\frac{d^2x'}{dt^2}-\frac{3(y-y')(x-x')}{r^5}\frac{d^2y'}{dt^2} \\
&\left.-\frac{3(z-z')(x-x')}{r^5}\frac{d^2z'}{dt^2}\right\}
\end{aligned}
$$

with similar expressions for $m\mathrm{Y}, m\mathrm{Z}$. . . . . (29)

22. From these equations it follows that, when two bodies are moving in the same direction through the æther, and

with the same unaccelerated velocity, the motional forces experienced by the bodies will be equal and opposite. They will not, however, have the same line of action, except when the motion is either in the line of centres or perpendicular thereto. When the common velocity $\mathbf{v}$ of the two bodies is *in the line of centres*, there is *repulsion* between the bodies amounting to

$$\frac{3}{2\pi}\frac{mm'\mathrm{F}^2\rho}{r^4}\mathbf{v}^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

When the common velocity $\mathbf{v}$ is *perpendicular to the line of centres* there is *attraction* amounting to

$$\frac{3}{4\pi}\cdot\frac{mm'\mathrm{F}^2\rho}{r^4}\mathbf{v}^2 = \mathrm{K}\frac{mm'}{r^4}\mathbf{v}^2, \quad . \quad . \quad . \quad . \quad (31)$$

where

$$\mathrm{K} \equiv \frac{3}{4\pi}\mathrm{F}^2\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

There is neither attraction nor repulsion when the common velocity of the two bodies is inclined at an angle $\tan^{-1}\sqrt{2}$, or approximately 54° 44′ to the line of centres. But in addition to the forces in the line of centres, there are in general forces perpendicular to this line, and if we imagine the two bodies connected by a rigid immaterial (or material) bar, the tendency will be to set the line of centres *perpendicular* to the direction of motion; the couple acting on the system only vanishing when the direction of motion is parallel or perpendicular to the line of centres. When the direction of motion is inclined 45° to the line of centres, the couple has its maximum value, namely,

$$\frac{3}{4\pi}\cdot\frac{mm'\mathrm{F}^2\rho}{r^3}\mathbf{v}^2 \quad \text{or} \quad \mathrm{K}\frac{mm'}{r^3}\mathbf{v}^2. \quad . \quad . \quad . \quad (33)$$

23. If we consider a bar made of given material and of definite proportions, and so suspended as to have a given period of oscillation about an axis through its centre of figure, then the angular deviation of the bar arising from a given velocity of translation through the æther will, at its maximum, be inversely proportional to the square of the linear dimensions of the bar. Though no experiments may have been specially made with the object of detecting motional forces of this kind, it seems certain that if more than a very slight

residual effect existed, it must have become apparent in other observations. None the less it would be interesting to make some definite experiments to put the question to the test, the apparatus required being simple. Incidentally, if the couples proved to be of measurable magnitude, we could readily obtain the data necessary for a determination of our motion relatively to the æther. This would involve no contradiction of the principle of relativity in electromagnetism; the phenomena with which we are here concerned not being electromagnetic in character.

24. Though we do not know what may be the value of the constant K defined by (32), or even whether it be different from zero, we can make some attempt to assign an upper limit to its value. Imagine two compact masses of 1 gram each, connected by a bar of negligible mass, the distance between their centres being 2 centimetres. Let this system be suspended by a quartz fibre, so that its axis is horizontal, and the period of its oscillations 30 seconds. Let us further suppose that we are able to detect any deviation as great as one second of arc on either side of a mean position, when the suspended system turns about a vertical axis. Now in virtue of the earth's orbital motion, the highest value attained by the velocity **v** must be at least as great as 32 kilometres per second ($=3{\cdot}2\times10^6$ cm. per second), so that by (33) our suspended system, when most favourably placed and oriented, could experience a couple as great as

$$\tfrac{1}{8}(3{\cdot}2\times10^6)^2\text{K} \doteqdot 1{\cdot}28\times10^{12}\text{K}. \quad . \quad . \quad . \quad (34)$$

Also the restoring moment when the system is displaced through one second of arc about a vertical axis is $4{\cdot}4\times10^{-7}$ dyne-cm.; and for the experiment to give a null result, we should accordingly have

$$\text{K} \not> 3{\cdot}4\times10^{-19} \text{ in C.G.S. measure.} \quad . \quad . \quad (35)$$

Assuming, in advance, that a null result would be obtained in this case, the value which, in the sequel, will be provisionally assumed is

$$\text{K}=2\times10^{-20}. \quad . \quad . \quad . \quad . \quad . \quad (36)$$

25. This tentative value of K may be compared with one suggested by another way of regarding the question. If the

negative electron could be regarded as having a vacuous spherical nucleus of radius 10^{-13} cm., and if we assume the atom of hydrogen to contain only one negative electron, while the number of molecules in 1 c.c. of gas at 0° C. and 1 atmosphere pressure is taken to be $3{\cdot}9 \times 10^{19}$; then it appears that, on the score of negative electrons alone, F, the extrusion of æther per unit mass of matter (§ 7), would be 4×10^{-15} c.c. of æther per gram of matter; equivalent to ·004 gram of æther per gram of matter, when an ætherial density of 10^{12} is assumed*. Though the existence of a vacuous nucleus in an electron does not appear reconcilable with some of the views adopted in this paper, it may be worth while to remark that the value of F just mentioned leads to $K = 4 \times 10^{-18}$, which is 200 times as great as the estimate (36), and about 12 times as great as might just suffice to give positive results in the hypothetical experiment of § 24.

Conjectural Estimates.

26. The values of a further number of physical magnitudes will have to be conjectured before a quantitative view of the theory can be attempted. Some of these magnitudes are introduced in the following seven paragraphs, which are devoted to establishing certain simple relations amongst the quantities concerned; the immediate object being to discover whether there is any consistent set of values which does not bring our theory into conflict with experience.

27. From (32)

$$F = 2\sqrt{\left(\frac{\pi K}{3\rho}\right)}. \quad \dots \quad (37)$$

As regards the primary disturbance, the problem (as in § 11) may be simplified by limiting our consideration to a single train of plane waves of definite wave-length λ, the greatest linear dimension of the region with which we are concerned being but a small fraction of λ, so that, to a sufficient degree of approximation, the pressure variations at any point within that region are given by

$$\mathfrak{p} = B \sin(2\pi nt - \epsilon), \quad \dots \quad (38)$$

* *Cf.* § 35 below.

n denoting the frequency of the disturbance. Equation (26) now becomes

$$G = \frac{H^2\rho}{8\pi} B^2 s^2 = \frac{\pi}{2} . H^2 \rho B^2 n^2, \quad . \quad . \quad . \quad (39)$$

since we are now limited to a single s, which is identical with $2\pi n$.

F, the extrusion of æther per unit mass of matter (§ 7) suffers variations given by

$$\delta F = -H\mathfrak{p} = -HB \sin (2\pi n t - \epsilon). \quad . \quad . \quad (40)$$

Let the greatest numerical value of δF be ζF; then by (40), (39),

$$\zeta^2 F^2 = H^2 B^2 = 2G/\pi\rho n^2; \quad . \quad . \quad . \quad . \quad (41)$$

or, remembering (37),

$$n = \frac{1}{\pi\zeta} \sqrt{\left(\frac{3G}{2K}\right)}. \quad . \quad . \quad . \quad . \quad . \quad (42)$$

28. Let κ denote the bulk-modulus of elasticity of the æther, or $1/\kappa$ its compressibility. By § 8 above, $H\sigma$ is the *additional* ætherial compressibility where there are σ grams of atomic matter per cubic centimetre; let this be equivalent to $\mu\sigma/\kappa$, so that $\mu\sigma$ is a pure number. Thus

$$H = \mu/\kappa. \quad . \quad . \quad . \quad . \quad . \quad (43)$$

In virtue of our assumption that κ is overwhelmingly greater than any other elastic modulus of the æther, we have for the square of the velocity of compressional waves

$$n^2\lambda^2 = \kappa/\rho,$$

whence

$$\kappa = n^2\lambda^2\rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

From (39), (43), (44),

$$B = \frac{n\lambda^2}{\mu} \sqrt{\left(\frac{2\rho G}{\pi}\right)}, \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$\frac{B}{\kappa} = \frac{1}{\mu n} \sqrt{\left(\frac{2G}{\pi\rho}\right)}, \quad . \quad . \quad . \quad . \quad . \quad (46)$$

and

$$\frac{B^2}{2\kappa} = \frac{\lambda^2}{\mu^2} \frac{G}{\pi E} . E, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where E is the constitutive energy per cubic centimetre of the æther. Thus the factor of E in the right-hand member

of (47) expresses the maximum potential energy per unit volume due to the wave-motion as a proportion of the constitutive energy of the medium. Moreover, this maximum potential energy per unit volume corresponds to an increment δF, equal to ζF, above the mean value F, and is thus, to our order of approximation, proportional to ζ^2. We may write, then,

$$\tfrac{1}{2}\zeta^2 = \vartheta \frac{\lambda^2}{\mu^2} \cdot \frac{G}{\pi E},$$

or

$$\zeta = \frac{\lambda}{\mu} \sqrt{\left(\frac{2G\vartheta}{\pi E}\right)}; \quad . \quad . \quad . \quad . \quad (48)$$

which serves to define a numerical constant ϑ now first introduced. This constant might, not improbably, be unity, or a number of that order of magnitude, and for illustrative purposes it is later assumed that

$$\vartheta = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

29. From (42), (48),

$$n = \frac{\mu}{2\lambda} \sqrt{\left(\frac{3E}{\pi K \vartheta}\right)}, \quad . \quad . \quad . \quad . \quad . \quad (50)$$

the velocity of compressional waves in free æther being

$$\mathfrak{V} = n\lambda = \frac{\mu}{2} \sqrt{\left(\frac{3E}{\pi K \vartheta}\right)}. \quad . \quad . \quad . \quad . \quad (51)$$

Using (50) in conjunction with (44) and (45) respectively,

$$\kappa = \frac{3\mu^2 \rho E}{4\pi K \vartheta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

$$B = \frac{\lambda}{\pi} \sqrt{\left(\frac{3EG\rho}{2K\vartheta}\right)}; \quad . \quad . \quad . \quad (53)$$

while from (43)

$$H = \frac{4\pi K \vartheta}{3\mu\rho E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

30. An expression is readily found for the maximum translational velocity attained by any element of æther in the primary wave-motion, half the square of this velocity multiplied by the density of the æther being equal to the maximum kinetic or maximum potential energy per unit volume; that is, equal to $B^2/2\kappa$. From (47) the desired expression is at once obtained, and only requires to be divided

by $2\pi n$ to give the semi-amplitude of vibration (maximum displacement), and multiplied by $2\pi n$ to give the maximum acceleration of any æther-element. Remembering (50) the three expressions are:—

$$\text{Maximum displacement from mean position} = \frac{\lambda^2}{\pi\mu^2}\sqrt{\left(\frac{2\mathrm{KG}\vartheta}{3\rho\mathrm{E}}\right)}, \quad (55)$$

$$\text{Maximum velocity} = \frac{\lambda}{\mu}\sqrt{\left(\frac{2\mathrm{G}}{\pi\rho}\right)}, \quad (56)$$

$$\text{Maximum acceleration} = \sqrt{\left(\frac{6\mathrm{GE}}{\vartheta\rho\mathrm{K}}\right)}. \quad (57)$$

31. Bearing in mind § 9, and in particular equation (3), we have also, *for any particle of neutral matter considered with respect to the æther*,

$$\text{Maximum displacement from mean position} = \frac{2\lambda^2\mathrm{K}}{3\mu^2}\sqrt{\left(\frac{2\vartheta\mathrm{G}}{\pi\mathrm{E}}\right)}, \quad (58)$$

$$\text{Maximum velocity} = \frac{2\lambda}{\mu}\sqrt{\left(\frac{2\mathrm{GK}}{3}\right)}, \quad (59)$$

$$\text{Maximum acceleration} = 2\sqrt{\left(\frac{2\pi\mathrm{GE}}{\vartheta}\right)}; \quad (60)$$

these three equations of course concern only that motion of matter with respect to the æther which is directly due to the primary wave-motion.

32. From (46), (50) we have for the range of proportional changes of volume of the æther, above and below mean value,

$$\frac{\mathrm{B}}{\kappa} = \frac{2\lambda}{\mu^2}\sqrt{\left(\frac{2\mathrm{GK}\vartheta}{3\rho\mathrm{E}}\right)}; \quad (61)$$

while for the rate (in ergs per second) at which wave-energy is being propagated through any square centimetre of surface whose plane is parallel to the wave-fronts, we must take the product of the wave-energy per c.c. multiplied by the velocity of propagation. Now throughout our purely progressive wave-train the *total* energy per unit volume is $\mathrm{B}^2/2\kappa$, the maximum value attained by the potential energy per unit volume. Thus multiplying (47) by (51) the rate at which wave-energy is being propagated, in ergs per second per

square centimetre, is found to be

$$\mho B^2/2\kappa = \frac{\lambda^2 G}{2\mu}\sqrt{\left(\frac{3E}{\pi^3 K\vartheta}\right)}. \quad . \quad . \quad . \quad (62)$$

33. All the quantities with which we are now concerned are expressed in terms of the following seven quantities:—

(i.) G the Newtonian gravitation constant $=6{\cdot}66\times10^{-8}$.
(ii.) K the motional constant, defined by (31) [2×10^{-20}].
(iii.) ρ the density of the æther [10^{12}].
(iv.) E the constitutive energy per c.c. of the æther [10^{33}].
(v.) λ the wave-length of the primary disturbance [10^4 astronomical units $=1{\cdot}5\times10^{17}$ cm.].
(vi.) μ the proportional increase of ætherial compressibility arising from the presence of 1 gram of atomic matter per c.c., as defined at the beginning of § 28 [20].
(vii.) ϑ a numeric defined by (48) [1].

Of these seven, only the first, the gravitation constant, is known, the values indicated (in square brackets) for the remaining six being merely conjectural. If no set of values could be found which did not lead to demonstrably false results, we should have to conclude that the theory in the form here suggested was untenable. The considerations leading to the choice of the values in question will be best understood after the corresponding values of some related quantities have been computed. The table (p. 92) gives in a collected form the relations obtained in §§ 27–32.

34. We may now review briefly the tentative values attributed in the last paragraph to K, ρ, E, λ, μ, and ϑ. The value 2×10^{-20} is assumed for the motional constant K, in accordance with (35); the supposition being that K is small enough for a null result to be obtained in the test suggested in § 24, but not with any great margin to spare. It might be that the test referred to would give a positive result, and in any case a much more sensitive disposition could easily be devised. But since no such positive result has yet been recorded, we cannot on this score assign any lower limit to the value of K, nor is our choice of something near the highest

Table.

Values of certain quantities connected with the hypothetical (primary) wave-motion, expressed in terms of G, K, ρ, E, λ, μ, and ϑ, together with the numerical values for the same quantities derived from those already assigned to G, K, &c.

Nature of quantity.		Symbol.	Equivalent expression.	Value in C.G.S. units.	Value otherwis expressed.
For any element of æther	Maximum displacement.	...	$\frac{\lambda^2}{\mu^2}\sqrt{\left(\frac{2\vartheta KG}{3\rho E}\right)}$	$6{\cdot}9\times10^{-5}$	·69 micron.
	Maximum velocity.	...	$\frac{\lambda}{\mu}\sqrt{\left(\frac{2G}{\pi\rho}\right)}$	$5{\cdot}6\times10^{6}$	56 kilom. per sec.
	Maximum acceleration.	...	$\sqrt{\left(\frac{6GE}{\vartheta\rho K}\right)}$	$4{\cdot}5\times10^{17}$	
For any element of atomic matter, with respect to the æther	Maximum displacement.	...	$\frac{2\lambda^2 K}{3\mu^2}\sqrt{\left(\frac{2\vartheta G}{\pi E}\right)}$	2×10^{-8}	1/5000 micron.
	Maximum velocity.	...	$\frac{2\lambda}{\mu}\sqrt{\left(\frac{2GK}{3}\right)}$	$1{\cdot}6\times10^{3}$	16 metres per sec
	Maximum acceleration.	...	$2\sqrt{\left(\frac{2\pi GE}{\vartheta}\right)}$	$1{\cdot}3\times10^{14}$	
Frequency of primary disturbance.		n	$\frac{\mu}{2\lambda}\sqrt{\left(\frac{3E}{\pi\vartheta K}\right)}$	$1{\cdot}29\times10^{16}$	
Velocity of compressional waves in æther.		$\mathfrak{V}$	$\frac{\mu}{2}\sqrt{\left(\frac{3E}{\pi\vartheta K}\right)}$	$2{\cdot}19\times10^{27}$	$2{\cdot}3\times10^{9}$ light-yea per second.
Extrusion of æther per unit mass of matter (§ 7).		F	$2\sqrt{\left(\frac{\pi K}{3\rho}\right)}$	$2{\cdot}9\times10^{-16}$	
Mass of æther extruded per unit mass of matter.		$F\rho$	$2\sqrt{\left(\frac{\pi\rho K}{3}\right)}$	$2{\cdot}9\times10^{-4}$	
Maximum proportional deviation of F from its mean value.		ζ	$\frac{\lambda}{\mu}\sqrt{\left(\frac{2\vartheta G}{\pi E}\right)}$	$5{\cdot}6\times10^{-5}$	
Bulk-modulus of elasticity of the æther.		κ	$\frac{3\mu^2\rho E}{4\pi\vartheta K}$	$4{\cdot}78\times10^{66}$	$4{\cdot}72\times10^{60}$ atmo-spheres.
Additional ætherial compressibility per unit density of matter present.		H	$\frac{4\pi\vartheta K}{3\mu\rho E}$	$4{\cdot}2\times10^{-66}$	
Maximum deviation of ætherial pressure from its mean value.		B	$\frac{\lambda}{\pi}\sqrt{\left(\frac{3EG\rho}{2\vartheta K}\right)}$	$3{\cdot}9\times10^{45}$	$3{\cdot}8\times10^{39}$ atmo-spheres.
Maximum proportional deviation of ætherial density from its mean value.		B/κ	$\frac{2\lambda}{\mu^2}\sqrt{\left(\frac{2\vartheta GK}{3\rho E}\right)}$	$8{\cdot}1\times10^{-22}$	
Total energy of primary disturbance per cubic centimetre.		$B^2/2\kappa$	$\frac{\lambda^2 G}{\pi\mu^2}$	$1{\cdot}6\times10^{24}$	$1/(6\times10^{8})$ of t constitutive ener of the medium.
Energy propagated through 1 sq. cm. of normal surface per second.		$\mathfrak{V}B^2/2\kappa$	$\frac{\lambda^2 G}{2\mu}\sqrt{\left(\frac{3E}{\pi^3\vartheta K}\right)}$	$3{\cdot}4\times10^{51}$ ergs.	$3{\cdot}4\times10^{41}$ kilowat per sq. cm.

of seemingly admissible values made necessary by the quantities given in the table. The experimental determination of the motional constant K, or of an upper limit to its value, would be interesting quite apart from its bearing on this theory; the question involved being, essentially, whether or not the total of ætherial substance comprised in any given volume is modified by the presence of atomic matter within that volume.

35. The values assumed for ρ and E (the density and the constitutive energy per unit volume of the æther) are those lately suggested by Sir O. Lodge *, who has given very convincingly his reasons for supposing that, in order of magnitude, the true values may not be very different from these estimates. Incidentally it may be remarked that Lodge's estimate of the constitutive energy per unit volume of the æther is about 6 times the electrostatic energy per unit volume, close to the surface of a negative electron, calculated on the usual assumptions that practically the whole inertia of the electron is electromagnetic, and that the ordinary linear relations of the electromagnetic field hold good right up to the surface of the electron.

36. For the wave-length of the primary disturbance, we must assume a value so great that, even at distances of at least several astronomical units, the inverse square law of gravitational attraction is sensibly accurate. By putting $\lambda = 10^4$ astronomical units, this condition seemed to be amply fulfilled, since for distances small compared with λ, the deviations from the inverse-square law would be only of the second order (§ 19). The chief consideration against assigning a very much greater value to λ is that ζ must be kept small to preserve linearity of relations, and also, I imagine, for general plausibility.

37. The same consideration (keeping ζ small) leads us to assign a fairly high value to μ (defined in § 28). It is difficult to see what sort of value μ should be expected to have: whether the proportional increase of ætherial compressibility due to a space being "filled" with water instead of being vacuous should be a large number or a small fraction. The value—20—chosen for μ would make the phase-difference

* Phil. Mag. vol. xiii., 1907; Nature, lxxv. p. 519, 1907.

of pressure-fluctuation of about the same order between the surface and the centre of the sun as between the surface of the sun and that of the earth, under maximum conditions.

The somewhat vague grounds on which the value unity is assigned to ϑ were indicated at the end of § 28.

38. Some of the derived quantities in the table above may now be considered. In the first place, take ζ, which represents the maximum deviation from its mean value of F the "specific displacement" of atomic matter (§ 7). If the primary disturbance is slight enough to be adequately represented by linear equations, we should expect on general grounds that ζ would be small, and the value for ζ derived from our assumptions is $5 \cdot 6 \times 10^{-5}$, while the energy of the primary disturbance per unit volume, under the same assumptions, appears as $1/(6 \times 10^{8})$ part of the constitutive energy of the æther.

39. For n, the frequency of the primary disturbance, $1 \cdot 29 \times 10^{10}$ per second is found, leading to $2 \cdot 19 \times 10^{27}$ cm. per second, or over two thousand million light-years per second, for $\mathfrak{V}$, the velocity of propagation of a compressional-rarefactional disturbance through the æther. This would amply suffice for the sensibly instantaneous character of gravitational attraction.

40. The bulk-modulus of elasticity of the æther, denoted by κ is, with our assumptions, represented by $4 \cdot 78 \times 10^{66}$ dynes per sq. cm., or $4 \cdot 72 \times 10^{60}$ atmospheres. Comparing this with the assumed constitutive energy of the æther (10^{33} ergs per c.c.), which is taken to be the basis of dielectric elasticity (the reciprocal of dielectric capacity) in free æther, the comparison appears to be consistent with our assumption that the bulk-modulus is enormously greater than any other elastic modulus of the æther; an assumption in accordance with which the æther, in relation to compressional-rarefactional motions, has been treated as a fluid.

Direct Effects of the Primary Disturbance.

41. Considering next the motion of the æther constituting the primary disturbance, and the directly resulting motion of matter with respect to the æther, the question arises: what phenomena, if any, would present themselves to our

observation if such motions were actually taking place? In the first place, bearing in mind § 12 above, it will be evident that a bodily vibratory motion of the æther, of sufficiently great wave-length, *with atomic matter equally partaking in such motion*, could give rise to no observable phenomena; if any effects are to be made manifest, as a direct consequence of the primary disturbance, these must arise from the motion of matter with respect to the æther. So far as I have been able to see, after long and careful consideration, there would be nothing capable of affecting interference phenomena, no heating effect, and no production of electromagnetic waves, provided only that one condition were realized. That condition is that the positive and negative electrons should have identical accelerations impressed on them by the direct action of the primary disturbance. The discrimination between positive and negative electrons in relation to gravitational agency is a question presenting many aspects for consideration. Some of these are touched upon in Appendices A and B.

42. Perhaps the most surprising of the tentative numerical values tabulated above, is that suggested as the rate at which energy is being propagated through the æther, per square centimetre of surface normal to the direction of propagation; the energy so propagated in one second exceeding by many million times the Sun's entire store of available heat. The problem has of course been simplified by limiting the primary disturbance to a single progressive wave-train of definite wave-length; but the result would not have been greatly different if a series of wave-lengths had been included as in § 18, and it is of course quite immaterial whether we assume the primary waves to be travelling indiscriminately in all directions, or predominantly or exclusively in a single direction. It must indeed be admitted that the basis of the estimates put forward in § 33 is nothing better than guess-work; but, after making all allowances for the very great uncertainty attaching to such conjectures, it appears to be an inseparable feature of the theory proposed that energy should be travelling through the æther on a prodigious scale.

43. The theory of gravitation which forms the subject of this paper has a good deal in common with its predecessors. In the first place, it is made to appear that gravitational attraction is not an essential and inseparable attribute of matter, the Newtonian constant being theoretically susceptible of increase or diminution, or even of entire suppression through a change of external circumstances. In this respect the present theory is comparable with Le Sage's hypothesis of ultramundane corpuscles, as well as with a scheme put forward by Challis *, which likewise assumed the existence of compressional waves in the æther, although the function attributed to these waves was very different. But the theory now advanced is much more akin to that of Prof. Hicks, which indeed suggested it, and of which it may be regarded as a development. The introduction of ætherial compressional waves of great wave-length to actuate the pulsatory movements of atomic matter, and thus secure the necessary agreement of phase amongst the pulsating centres, is one of the main modifications suggested; another being the manner in which the capacity for pulsatory motion, assumed to be associated with the electrons, is represented as mobile through the æther. In this way we avoid the necessity for supposing that any element of ætherial substance ever deviates by more than a minute amount from its mean position, the motion throughout being of "stationary" type, while at the same time the problem with which we have to deal is essentially one of hydrodynamics, capable of being worked out to a first order by means of a simple analysis. We are enabled to treat the æther as if it were a fluid, not because we assume it to yield freely to certain types of stress, but because the distortions involved in the motions considered are so excessively minute that no appreciable opposing stresses are called up, save only the changes of hydrostatic pressure, which owe their importance to the enormous value attributed to the bulk-modulus of elasticity of the æther. Apart from the simplicity thus attained, it appears to me most desirable that we should discard, if possible, the conception of matter, or of

* See a review by Maxwell, 'Nature,' vol. viii.; Scientific Papers, vol. ii. p. 338.

any parts or properties associated with matter, as grossly ploughing a course through a reluctant æther. All that we know of æther and of matter seems to indicate that the mobility of matter is absolute, and that the elastic properties of the æther remain perfect, notwithstanding the motions of material bodies, or even of detached negative electrons travelling with velocities approaching that of radiation. And the only way in which I have been able to conceive of matter as travelling through the æther, without doing unwarrantable violence to the structure of the medium, is by supposing the entire phenomenon of motion to be reducible to a transference of strain, so that no event in our universe involves a progressive yielding of the æther.

44. By Maxwell * it was felt to be an objection to such hypotheses as those of Le Sage and of Challis, that they involved a continual expenditure of energy for the maintenance of gravitational attraction, the conservation of energy in such cases being "apparent only." The present theory is equally open to this criticism ; but the objection is a metaphysical rather than a physical one, and in view of recent developments it has hardly the force which it might have appeared to have some forty years ago. We have become accustomed to the idea that our rapid motion through a medium of enormous density not only fails to provide us with a useful source of energy, but defies the most refined attempts to detect it by means of terrestrial observations.

And there is nothing more inherently improbable in the notion that our universe may be traversed by waves of enormous energy, perceptible to us only by means of a minute secondary effect—gravitation. That we should be unable, even in our dealings with this feeble residual phenomenon, to extract with continual profit the minutest portion of the energy so abundantly propagated, is a view which may appear to us somewhat ironical, but which is not out of harmony with the trend of modern physical conceptions.

45. Whatever may be the difficulties of the theory, this attempt to contribute something towards the explanation of gravitation has appeared to me sufficiently suggestive to be

* *Loc. cit.*

worth publication; the more so as incidentally certain questions are raised, some of which, it may be hoped, are capable of being decided by experiment.

Appendix A.

Electromagnetic Phenomena which might conceivably arise from Gravitation.

46. Since in this paper gravitation is regarded as a secondary or residual effect, due to the influence of atomic matter on the propagation of compressional ætherial waves, it might have seemed more logical to discuss the electromagnetic phenomena possibly produced by such waves, before considering effects of a like character which might be supposed to arise from gravitational attraction. In the first place, however, gravitation is known to exist, and in the second place, the effects to be considered, being statical, are comparatively simple. Accordingly gravitation may be dealt with first in this connexion.

47. Independently of any theory as to the cause of gravitation, if we suppose that ordinary matter is made up of electrons, then it seems reasonable to conclude that any influence exerted on matter can be analysed into influences exerted on electrons. Again, experiment indicates unmistakably that, whatever the nature of positive and negative electrons may be, they are far from being symmetrical opposites. Thus it is natural to inquire what effects are to be expected when we discriminate between positive and negative electrons under gravitational influence.

48. To take the simplest case, consider a body moving without constraint in a uniform field of gravity, the acceleration of the body f thus agreeing in direction and magnitude with the strength of the field. Suppose that, electrically, the condition of the body is not changing; then on the whole both positive and negative electrons are moving with the acceleration f. Now if the forces exerted in a field of gravity are in the same sense on positive and negative electrons, and of magnitudes proportional respectively to the masses of such electrons, the identical acceleration of the two denominations will follow, without additional forces of electrostatic type

being called into play. But if the gravitational forces on positive and negative electrons do not conform to the condition just referred to, they can always be resolved into two sets: one set of like forces proportional respectively to the masses of the positive and negative electrons on which they act, and another set of forces acting equally and oppositely on the positive and the negative. This latter set produces on the body under consideration the seme effect as would result from a uniform electrostatic field, the body becoming electrically polarized if of dielectric material, or, if a conductor, acquiring a surface charge of electrification without internal polarization.

49. The state of things here suggested is somewhat different from anything ordinarily contemplated in electrostatics; the gravitational quasi-electromagnetic intensity which acts throughout the substance of a conducting body being balanced by the true electromotive intensity arising from the surface distribution of electricity, so that we have an electrostatic field of force exerted in a conducting medium in equilibrium. In these circumstances, it may not be superfluous to point out that, in estimating the electric field-intensity from the surface charges, no dielectric constant other than that of free æther comes into play. For in an electrostatic field, the intensity at any point is determined jointly by the signs and positions of all the electrons concerned, and by nothing else, each electron contributing to the field-intensity the same component as if the other electrons were non-existent. As is well understood, the electronic theory affords on these lines an account of the dielectric qualities of different media; such qualities, however, being without any bearing on the problem, so long as *on the whole* there are no forces tending to displace relatively to one another the positive and the negative electrons which together make up the medium under consideration.

50. Let μ_1, μ_2 denote the *masses* of the positive and negative electrons respectively, contained in unit mass of matter; then

$$\mu_1+\mu_2=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63).$$

Also let the total *charge* of the positive electrons in unit

mass of matter be $\epsilon\chi$, the total charge of the negative electrons being accordingly $-\epsilon\chi$: where ϵ is the quantity of electricity required to liberate one gram of hydrogen electrolytically, and χ is a numeric.

Further, in a field of gravity whose intensity is measured by the acceleration f, let

gravitational attraction on mass μ_1 *of positive electrons be* $\mu_1 \gamma_1 f$,
  ,, ,, ,, μ_2 *of negative electrons be* $\mu_2\gamma_2 f$.

We then readily find, for a mass moving freely in the field f, that the electrical condition is the same as if an electrostatic field were exerted, whose intensity, measured in the same direction as f, is

$$(-\mu_1+\mu_1\gamma_1)f/\epsilon\chi=(\mu_2-\mu_2\gamma_2)f/\epsilon\chi\,; \qquad (64)$$

this equivalent field-intensity being expressed in electrostatic or electromagnetic measure, according as ϵ is in electrostatic or electromagnetic measure.

51. The (single) condition that this quasi-electromotive-intensity should vanish is $\gamma_1=1=\gamma_2$, in which case the gravitational forces acting on masses μ_1, μ_2 of positive and negative electrons in a field of intensity f are respectively $\mu_1 f$, $\mu_2 f$; that is to say, in the proportion of the masses. As will be seen in Appendix B, the gravitation theory of this paper appears almost certainly to demand that the condition just referred to should be satisfied; but putting that theory for the time being on one side, it is interesting to trace out some consequences of a less restricted view as to the relative behaviour of positive and negative electrons under the action of gravity.

52. When the body with which we have to deal is not moving freely in a sensibly uniform gravitational field, the corresponding results cannot be written down without making some assumption as to the relative mobility of positive and negative electrons in a conducting or dielectric mass. For example, let us trace the consequences of supposing that the negative electrons alone possess sufficient mobility to enable them to migrate through unrestricted distances within the mass of a solid conducting substance; the negative electrons thus serving to convey the whole of any current which may

be flowing in the substance. In a cylindrical conductor whose axis is vertical, let a current be flowing upward; consider what happens when negative electrons whose aggregate charge is $-\epsilon\chi$ pass downward through the cylinder, and in particular through a certain stratum of the cylinder, bounded above and below by horizontal planes, whose distance apart is h. According to our notation, $-\epsilon\chi$ is the aggregate charge of the negative electrons which are comprised in one gram of neutral matter, and which have collectively the mass μ_2, experiencing in a gravitational field of intensity g the force $\mu_2\gamma_2 g$, measured downward, so that the work done by gravity while the quantity $-\epsilon\chi$ moves downward through the stratum h is $\mu_2\gamma_2 gh$. There is thus virtually, corresponding to a height h of the conductor, an electromotive force

$$-\mu_2\gamma_2 gh/\epsilon\chi \text{ (measured downward)} ; \quad . \quad . \quad (65)$$

the quasi-electromotive-intensity due to the action of gravity being therefore

$$-\mu_2\gamma_2 g/\epsilon\chi \text{ (measured downward)}. \quad . \quad . \quad . \quad (66)$$

53. If, in place of gravity acting, an acceleration f is impressed on an isolated conductor, then the negative electrons of total charge $-\epsilon\chi$, having an aggregate mass μ_2, must experience, when there is electrical equilibrium, a resultant force $\mu_2 f$, and in virtue of the assumed mobility of the negative electrons, this resultant force $\mu_2 f$ must be due to an electrical distribution on the conductor in question. But an electrical distribution which would exert the force $\mu_2 f$ on certain electrons is precisely that distribution which would arise from such an external field as would by itself exert the force $-\mu_2 f$ on the same electrons. In other words, the conducting body caused to move with acceleration f, becomes electrified precisely as if it were placed in an electrostatic field of intensity

$$\mu_2 f/\epsilon\chi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

If we take f to be vertically downward, and identical in value with g, we find by combining (66) and (67) an expression for the virtual electrostatic field due to the action of gravity on a freely falling body; and this expression agrees

with (64) which was obtained independently of any assumption regarding the respective mobilities of positive and negative electrons.

54. On the other hand, if we were to assume that, while currents of conduction were carried exclusively by the negative electrons, it was the positive electrons alone which were acted upon by gravitational attraction, we should be led to the conclusion that bodies at rest relatively to (say) the earth would acquire no electrical charges through the action of gravity.

55. Reverting now to the illustrative assumption of § 52, we may attempt to form some idea of the magnitude of the electromotive effects to be expected when the negative electrons are supposed to be not only the exclusive carriers of currents of conduction, but also the only objects influenced by gravity. A further assumption is needed as to the total of positive or of negative charges carried by the electrons in unit mass of matter. Let it be assumed, for example, that $\chi=1$ (see § 50), so that the negative electrons comprised in one gram of hydrogen (or of other substance) will have an aggregate charge equal to ϵ, the quantity of electricity required to liberate one gram of hydrogen by electrolysis. The expression (66) for the downwardly-directed quasi-electromotive intensity resulting from the direct action of gravity becomes $-\mu_2\gamma_2 g/\epsilon$, where $\mu_2\gamma_2 g$ is the downward force exerted in the earth's gravitational field upon the mass μ_2 of negative electrons comprised in one gram of matter. But under our present assumptions this latter force is simply the weight of one gram of matter, and is equivalent to g; so that finally the downwardly-directed quasi-electromotive intensity is measured by $-g/\epsilon$. Now ϵ is roughly 10^4 e.m. units of quantity per gram, and g may be taken as about 980 cm./sec.^{2}; consequently, for the quasi-electromotive intensity affecting stationary bodies at the earth's surface the estimate obtained is 980×10^{-4} e. m. units of potential per cm., or $9{\cdot}8 \times 10^{-10}$ volt per cm.

56. On applying the axiom that gravity cannot be made to furnish an unlimited supply of energy, it is evident that the total gravitational electromotive force round any closed circuit of conducting bodies must vanish, however small may

be the conductivity of those bodies; so that any attempt to detect such electromotive effects galvanometrically must fail. The only methods conceivably available would be those depending upon the convection of electric charges by insulated conductors moving between one level and another, and making contact intermittently with the extremities of an elongated vertical conductor. It would be easy to devise a machine for performing continually a cycle of operations, whereby any electromotive influence due to gravity would be rendered effective in gradually imparting contrary charges to a pair of insulated hollow conductors. But apart from the excessive minuteness of the effect to be looked for, and the thermoelectric and other disturbing influences almost necessarily encountered, another consideration steps in to render any such attempt nugatory. To guard against stray electric influences, our whole apparatus would require to be enclosed in a conducting envelope, the charge induced on which by gravitational influence would exactly neutralize, throughout the interior of the envelope, the quasi-electromotive intensity due to the direct action of gravity. Thus, even if we suppose positive and negative electrons to be oppositely acted upon by a gravitational field (in which case, for example, the aggrgate attractive force on the negative electrons in a body might greatly exceed the weight of the body), it would still be impossible, by means of laboratory experiments, to detect any electromotive effects due to the earth's gravity.

57. The case of the earth moving in its orbit under the sun's attraction may be regarded as that of a body moving freely under a sensible uniform field of gravity. Making any assumptions that suggest themselves as to the number of electrons in a gram of matter, the relative masses of positive and negative electrons, and the forces experienced by these under gravitational influence (64), may be used to estimate the resulting electrical distribution on the earth's surface (or in the upper strata of the atmosphere) arising directly from the gravitational attraction of the sun. But with such assumptions as I have tried, it appears that the effect to be expected is very minute. For example, making the same assumptions as in § 52, and in addition assuming that nearly all the inertia of neutral matter is the

inertia of positive electrons, the extreme difference of potential between opposite poles of the earth comes out as about 80 microvolts. Moreover, the shifting of this feeble electrical distribution, owing to the earth's (solar) diurnal rotation, could not have any sensible influence on the phenomena of terrestrial magnetism.

APPENDIX B.

Electromagnetic and Thermal Effects produced by Compressional Ætherial Waves.

58. Returning more particularly to the view adopted in the body of this paper, the results obtained in §§ 16–18 above may be readily revised so as to discriminate between the possibly different gravitative behaviour of positive and negative electrons.

As in Appendix A, let unit mass of neutral matter contain mass μ_1 of positive and μ_2 of negative electrons (so that $\mu_1+\mu_2=1$), the aggregate charge of mass μ_1 (or μ_2) of positive (or negative) electrons being as before denoted by $\epsilon\chi$ (or $-\epsilon\chi$).

Let

volume of æther "extruded" by mass μ_1 of positive electrons $=F_1\mu_1$,

" " " " μ_2 " negative " $={}_2\mu_2$;

then

mass " " " μ_1 " positive " $=F_1\mu_1\rho$,

and

mass " " " μ_2 " negative " $=F_2\mu_2\rho$.

Also let $\qquad -dF_1/dp=H_1, \quad -dF_2/dp=H_2; \qquad . \quad . \quad (68)$

then evidently

$$F_1\mu_1+F_2\mu_2=F(\mu_1+\mu_2)=F \quad \text{and} \quad H_1\mu_1+H_2\mu_2=H. \qquad (69)$$

Precisely as in (7), we can write down, in terms of the acceleration of the æther, the corresponding forces on positive and negative electrons. Writing A for the acceleration of the æther at any instant, arising from the propagation of the primary waves, the corresponding force tending to accelerate the mass μ_1 of positive electrons relatively to the æther is $F_1\mu_1\rho A$, while the mass μ_2 of negative electrons

similarly experiences a force $F_2\mu_2\rho A$. Now

$$\left.\begin{array}{l} F_1\mu_1\rho A = A\{F\mu_1\rho + (F_1 - F)\mu_1\rho\} \\ F_2\mu_2\rho A = A\{F\mu_2\rho + (F_2 - F)\mu_2\rho\} \end{array}\right\} \quad . \quad . \quad (70),$$

the aggregate forces acting respectively on the positive and on the negative electrons in unit mass of matter being thus resolvable into forces $F\mu_1\rho A$, $F\mu_2\rho A$ proportional to the masses on which they act, together with the pair of forces $(F_1 - F)\mu_1\rho A$ and $(F_2 - F)\mu_2\rho A$, which by (69) are equal and opposite. The first two constituents named tend to accelerate each element of matter with respect to the æther as a whole, the total force exerted per unit mass of matter being $F\mu A$; while the remaining equal and opposite forces, acting respectively on positive and negative electrons whose aggregate charges are $\epsilon\chi$, $-\epsilon\chi$, have the same effect on the motion of those electrons as would be produced by an electromotive intensity

$$T = \frac{(F_1 - F)\mu_1\rho A}{\epsilon\chi} = \frac{(F - F_2)\mu_2\rho A}{\epsilon\chi}, \quad . \quad . \quad (71)$$

measured in the direction of the acceleration A.

59. As in Appendix A, let it be assumed (merely by way of example) that $\chi = 1$, ϵ being as before the quantity of electricity required to liberate one gram of hydrogen electrolytically. Assume also that the extrusion of æther F per unit mass of neutral matter is due entirely to negative electrons; the æther extruded by one gram of matter being therefore that extruded by a mass μ_2 of negative electrons, whose total charge is $-\epsilon\chi$ $(= -\epsilon)$. Assume, that is to say,

$$F_1 = 0 \quad \text{and} \quad F_2\mu_2 = F\mu = F(\mu_1 + \mu_2). \quad . \quad . \quad (72)$$

If we further assume (as in Appendix A, § 57) that nearly all the inertia of neutral matter is the inertia of positive electrons, so that $\mu_1 = 1$ approximately, then (71) becomes

$$T = -\frac{F\rho A}{\epsilon}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

which, with our present special assumptions, is the quasi-electromotive intensity due to an acceleration A of the æther.

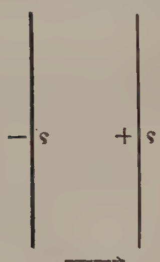
60. As regards the thermal effects arising from compressional waves in the æther, let us simplify the problem to

be considered by reducing the disturbance (as in § 27) to a single plane wave-train of very great wave-length λ, and of frequency n. Putting s in place of $2\pi n$, we may write for any region of dimensions very small compared with λ,

$$T = T_0 e^{ist}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

where it is to be understood that T and T_0 are expressed in absolute *electromagnetic* measure.

61. Consider in particular the case of a flat parallel-faced slab of conducting material, whose length and breadth are very great in comparison with its thickness, the main faces of the slab being perpendicular to the direction in which the compressional ætherial waves are being propagated, and perpendicular therefore to the direction of the quasi-electromotive intensity T, which we shall suppose to be measured positively from left to right in the figure, as indicated by the arrow.



At any instant let ς be (in electromagnetic measure) the surface-density of electrification on the right-hand face, $-\varsigma$ being that on the left-hand face; ς will evidently be a periodic function of the time of the form

$$\varsigma = \varsigma_0 e^{ist}, \quad . \quad . \quad . \quad . \quad (75)$$

where ς_0 is a constant, in general complex.

Between the faces of the slab there is, at any time t, a conduction current whose density, referred to unit of area, is $d\varsigma/di$, and a polarization current whose density is $-d\varsigma/dt$ *, the total effective current being thus zero. The same result is otherwise evident; for at any point exterior to the slab the total electromotive intensity arising from the surface-densities ς and $-\varsigma$ vanishes at each instant. Hence outside the slab there is no polarization and no polarization-current, and of course no conduction-current; so that the total effective current is zero outside the slab. The solenoidal or "stream" character of the total current accordingly necessitates a zero value for this vector within the substance of the slab also.

* *Cf.* Appendix A, § 49.

62. Thus everywhere, except near the edges of the slab (which we ignore) magnetic force is absent, and electromagnetically speaking our system is without kinetic energy *. There must of course be some kinetic energy involved in the reciprocating relative motions of positive and negative electrons, but this kinetic energy, even though a close enough analysis would reveal its electromagnetic character, has no place in the ordinary equations of electromagnetism, and a moment's consideration suffices to indicate its excessive minuteness.

63. Within the substance of the slab there is at each instant a quasi-electromotive intensity T and a true electromotive intensity $-4\pi V^2\varsigma$, V being the velocity of radiation. For simplicity it will now be supposed that the only opposing forces are due to electrical resistance; thus in the absence of appreciable inertia effects, the relation

$$(T-4\pi V^2\varsigma)c=d\varsigma/dt \quad . \quad . \quad . \quad . \quad . \quad (76)$$

is always satisfied; c being the conductivity of the substance of the slab. Using (74) and (75),

$$\varsigma_0=T_0c/(4\pi V^2c+is). \quad . \quad . \quad . \quad . \quad . \quad (77)$$

This gives for the current-density

$$I=is\varsigma_0e^{ist}=\frac{Ac(s^2+i\,.\,4\pi V^2sc)}{16\pi^2V^4c^2+s^2}\,e^{ist}; \quad . \quad . \quad (78)$$

or, separating the real from the imaginary problem, we have, corresponding to

$$T=T_0\cos st, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

$$I=\frac{T_0cs}{16\pi^2V^4c^2+s^2}(s\cos st-4\pi V^2c\sin st).. \quad . \quad (80)$$

* There will accordingly be no production of electromagnetic waves, except near the edges of the slab where the same simple conditions are not realized. But *in general* electromagnetic waves will be produced wherever there are bodies of dielectric or conducting material in the path of the primary compressional waves. If the primary disturbance, as in the foregoing table, is assumed to be periodic and of frequency $1{\cdot}29\times10^{10}$, the electromagnetic waves, being of the same frequency, will in free æther have a wave-length of 2·33 cm. Without having considered these waves in any detail, I think it may be said that they will be quite insignificant, provided the electromotive effects of the primary waves are too slight to give rise to appreciable thermal phenomena.

64. The rate, in ergs per second, at which energy is being dissipated as heat in each cubic centimetre of the conductor is accordingly

$$\frac{I^2}{c}=\frac{T_0^2cs^2}{16\pi^2V^4c^2+s^2}(s\cos st-4\pi V^2c\sin st)^2; \quad . \quad (81)$$

so that

$$\text{average of }\frac{I^2}{c}=\frac{1}{2}\frac{T_0^2cs^2}{16\pi^2V^4c^2+s^2}. \quad . \quad . \quad . \quad (82)$$

The last-written expression is a maximum with respect to c when $c=s/4\pi V^2$, in which case the average of I^2/c becomes

$$T_0^2s/16\pi V^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

64 *a*. Remembering (73), we find that, with the particular set of numerical values adopted in the foregoing table, $T_0=130$ volts per cm., or in absolute e.m. units $1{\cdot}3\times10^{10}$; $s=2\pi n=2\pi\times1{\cdot}29\times10^{10}$; the average rate at which energy is being dissipated in the conductor being in that case

2×10^8 *ergs per sec. per c.c.* or about $7{\cdot}5$ *calories per sec. per c.c.*

This is with the conductivity most favourable to dissipation, the value of the conductivity being then given by $c=s/4\pi V^2$; so that the specific resistance would be $4\pi V^2/s=1{\cdot}4\times10^{11}$ in absolute measure, which is equivalent to 140 ohms as the resistance of a centimetre cube. If the slab were of much higher or much lower conductivity, the dissipation of energy (corresponding to the same set of assumptions) would of course be much less.

65. Thus with the special assumptions of § 59 the rate of development of heat, through the action of the primary waves on matter, though excessively minute compared with the rate at which we have supposed energy to be propagated in those waves, is nevertheless, at its maximum, much greater than could be reconciled with experience. But in obtaining the estimates of the previous paragraph, it was assumed that the "extrusion" of the æther per unit mass of matter * was entirely due to the negative electrons, and that the inertia of matter was almost entirely due to the positive electrons. Unless we were to suppose the extrusions due to

* *Cf.* § 7.

positive and negative electrons to be of opposite signs, this would be the assumption corresponding with the greatest electromotive effects from the action of the primary waves on atomic matter, and with the greatest consequent dissipation of energy ; the inference being that, if we are to retain our tabulated estimates, or any others not wholly different from these in order of magnitude, we must adopt some more favourable assumption as to the relative "extrusions" and masses of positive and negative electrons. Now it is evident from (71) that if the "extrusions" of positive and negative electrons were proportional to their masses (that is $F_1 = F = F_2$), the electromotive effects of the primary waves would vanish. In that case there would be no heating of matter through dissipation of the energy of the primary disturbance, nor would there be any corresponding production of electromagnetic waves. Experience suggests, then, that the relation

$$F_1 = F = F_2 \qquad \dots\dots\dots \qquad (84)$$

is exactly or very nearly fulfilled.

66. If indeed the æther is traversed by waves of other than electromagnetic type, to the penetration of which no sort of matter forms a barrier, it would be interesting to test for any residual heating effect due to the interaction of such waves with atomic matter. To devise the most sensitive arrangement within given limits of size wound need some thought, but evidently a body of large bulk would be best. Such a body (perhaps hollow) of suitable conductivity, well jacketed with a material of the highest possible thermal and electrical resistance, and further enveloped in a double-walled vessel containing ice, might be examined for a gradual rise of temperature. Even if all laboratory experiments on these lines should give negative results, it might still be held possible that some part of the earth's heat was due to the electromotive effect of compressional ætherial waves. This is only mentioned as a possibility suggested by the theory of this paper, and not as tending to remove any existing difficulty in the domain of geophysics.

67. Some attempt having already been made in Appendix A to trace the possible electromotive effects of a gravitational

field, the modifications which must be made in the equations of § 17 above, when we wish to discriminate between positive and negative electrons, will now be very briefly indicated. With the notation explained in § 58, the source whose strength is expressed by (10) must now be represented by the more complete expression

$$-(\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2)\frac{\partial \mathfrak{p}}{\partial t}\sigma\, dx\, dy\, dz, \quad . \quad (85)$$

which is to be equated to $\nabla^2\phi\, dx\, dy\, dz$; hence (11) becomes

$$\phi=\frac{\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2}{4\pi}\frac{\partial \mathfrak{p}}{\partial t}\iiint\frac{\sigma'}{r}dx'\, dy'\, dz'; \quad . \quad . \quad (86)$$

while (16) is replaced by

$$\frac{\partial \pi}{\partial x}=\frac{(\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2)\rho}{4\pi}\frac{\partial^2 \mathfrak{p}}{\partial t^2}\mathrm{I}_x, \quad . \quad . \quad . \quad . \quad (87)$$

where I_x, as before, has the meaning indicated by (17).

In place of (18) we have now two equations, expressing the forces acting on positive and negative electrons respectively. The force on the positive electrons in unit mass of matter

$$=-\mu_1\mathrm{F}_1\frac{\partial \varpi}{\partial x}=-\mu_1\mathrm{F}_1\frac{(\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2)\rho}{4\pi}\frac{\partial^2 \mathfrak{p}}{\partial t^2}\mathrm{I}_x, \quad . \quad (88)$$

the force on the negative electrons being similarly

$$=-\mu_2\mathrm{F}_2\frac{\partial \varpi}{\partial x}=-\mu_2\mathrm{F}_2\frac{(\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2)\rho}{4\pi}\frac{\partial^2 \mathfrak{p}}{\partial t^2}\mathrm{I}_x; \quad . \quad (89)$$

(19) being replaced by

$$\left.\begin{array}{l}\mathrm{F}_1=\overline{\mathrm{F}}_1+\delta\mathrm{F}_1=\overline{\mathrm{F}}_1-\mathrm{H}_1\mathfrak{p}\\ \mathrm{F}_2=\overline{\mathrm{F}}_2+\delta\mathrm{F}_2=\overline{\mathrm{F}}_2-\mathrm{H}_2\mathfrak{p}\end{array}\right\}. \quad . \quad . \quad . \quad . \quad (90)$$

Hence the gravitational force acting on the positive electrons in unit mass of matter is

$$-\frac{\rho}{4\pi}\mu_1(\overline{\mathrm{F}}_1-\mathrm{H}_1\mathfrak{p})(\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2)\frac{\partial^2 \mathfrak{p}}{\partial t^2}\mathrm{I}_x; \quad . \quad (91)$$

and since the average value of $\partial^2\mathfrak{p}/\partial t^2$ is zero, the only part of this expression which does not disappear on averaging is

$$\text{average of }\left\{\frac{\rho}{4\pi}\mathrm{H}_1\mu_1(\mathrm{H}_1\mu_1+\mathrm{H}_2\mu_2)\mathfrak{p}\frac{\partial^2 \mathfrak{p}}{\partial t^2}\mathrm{I}_x\right\}. \quad . \quad (92)$$

Similarly, the average force on the negative electrons in unit mass of matter is

$$\text{average of } \left\{ \frac{\rho}{4\pi} H_2\mu_2(H_1\mu_1 + H_2\mu_2)\mathfrak{p}\frac{\partial^2\mathfrak{p}}{\partial t^2} I_x \right\}. \quad . \quad (93)$$

On adding (92), (93) and comparing with (21), it is seen that $(H_1\mu_1 + H_2\mu_2)^2 = H^2$, which is otherwise immediately evident.

68. Now we have seen that the only escape from inadmissibly great heating effects of the primary waves lies in supposing the relation (84) to be at least very approximately true; and unless we make the very special assumption that the relation in question is only fulfilled for one particular value of the general ætherial pressure, we must likewise conclude that

$$dF_1/dp = dF/dp = dF_2/dp; \quad . \quad . \quad . \quad . \quad (94)$$

or, remembering (68), that

$$H_1 = H = H_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

In this case the expressions (92), (93) for the forces exerted in a given gravitational field on the positive and the negative electrons in unit mass of matter, become proportional respectively to the aggregate masses of the electrons in question. As was pointed out in § 51 (Appendix A), this is the condition that no electromotive effects shall be experienced by a body moving freely under a sensibly uniform field of gravity.

The relation between the notation of this Appendix and that of Appendix A is evident: μ_1 and μ_2 having the same meaning in each case, while H_1, H_2 are respectively equivalent to $\gamma_1 H$, $\gamma_2 H$.

Appendix C.

A Non-Electromagnetic Term in the Inertia of an Electron.

69. It will readily be realised that, just as an electron possesses mass or inertia of electromagnetic origin, so on our theory the total effective mass of the electron comprises what may be called a gravitational term. The motions which are, on this view, to be classed as of gravitational type are wholly

of the nature of irrotational bodily movements of the volume-elements of æther. What has here been called the primary disturbance is assumed to consist of compressional waves, while the secondary disturbance—the supposed cause of gravitation—takes the form of a pulsatory movement whereof every particle of matter may be regarded as a centre. Finally, the motion of matter with respect to the æther appears in general to involve (over and above electromagnetic phenomena) an irrotational distribution of ætherial motion. This necessarily implies the addition of a corresponding term to the inertia of the matter in question; but as will now appear this term may, on our assumptions, be relatively insignificant. By way of illustration, expressions for the gravitational inertia of an electron, obtained on two further alternative assumptions, will now be given.

70. In the first place, let us suppose that the free æther is a strictly continuous medium, and that the constitution of an electron involves a modification of ætherial density expressible as a continuous function of coordinates. Let the modification of density be such as corresponds with a radial displacement δR (measured outwards) of any element distant R from the centre of the electron. It is understood, of course, that the unmodified state of the æther, expressed by $\delta R=0$, is one of uniform density. In particular, let *

$$\left.\begin{array}{lll} \delta R = CR & \text{from} \quad R=0 & \text{to} \quad R=R \\ \delta R = CR_1^3/R^2 & \text{from} \quad R=R_1 & \text{to} \quad R=\infty \end{array}\right\}; \quad . \quad (96)$$

then it can be shown that the corresponding term in the inertia of the electron is

$$\frac{20}{3}\pi\rho C^2 R_1^3, \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

where ρ as before is the density of the æther.

* This distribution of δR suffers from the disadvantage that motion of the electron through the æther involves impulsive changes of velocity of those æther elements for which, instantaneously, $R=R_1$. A slight modification of (96) would remove the objection, but at a great sacrifice of that analytical simplicity which must always be a leading consideration in the choice of purely illustrative examples.

71. Now the mass of æther "extruded" by the electron is $4\pi\rho CR_1^3$ and the expression (97) bears to this the ratio $\frac{5}{3}C$, which may be very small provided C is small enough. Moreover, in the table given above, the mass of æther "extruded" by one gram of neutral matter is estimated at $2{\cdot}9 \times 10^{-4}$ gram; and it therefore seems possible that the theory proposed may involve only a very minute non-electromagnetic term in the total inertia of a body.

72. Alternatively let the æther be regarded (for present purposes of illustration) as made up of thin coreless vortex-filaments pervading a frictionless liquid * of density ρ; the sum total of the vacuous cores comprised in any considerable volume of æther being so small a proportion of that volume that ρ may sensibly be identified with the average density of the æther. Let us further suppose, as in § 70, that the constitution of an electron involves ætherial displacements expressible by (96), and that such differences of density as exist from one volume-element of æther to another are due solely to altered diameter of the vacuous cores, no alteration in the total length of cores per unit volume being involved. It then appears that what we have called the gravitational part of the inertia of an electron, in addition to the quantity (97) contains a term

$$\tfrac{2}{3}\rho \times \text{volume of electron} \times \text{average value of} \left\{\text{volume of vacua per unit volume of æther} \times \left(\frac{d\delta r_0}{dR}\right)^2 \log\frac{r_1}{r_0}\right\}; \quad . \quad (98)$$

where $r_0 + \delta r_0$ is the radius of a vacuous core at a distance R from the centre of the electron, and r_1 is a length corresponding in order of magnitude with a scale of structure of the æther. In (98) "volume of electron" must be understood as extending to all that region wherein $d\delta r_0/dR$ differs sensibly from zero. The addition of the term (98) does not disturb the conclusion that the gravitational part of the inertia of an electron may be very small compared with the mass of æther which the electron "extrudes."

* *Cf.* Appendix D.

Appendix D.

A Kinetic Model of a Slightly Compressible Medium.

73. The representation of the electromagnetic properties even of free æther by means of a turbulently moving liquid is beset by difficulties which may well prove to be insuperable; but it is nevertheless interesting to remark that a perfectly incompressible liquid, through which vortices are distributed, constitutes a kinetic model of a slightly compressible medium such as we have assumed the æther to be, provided some or all of the vortices are coreless. One way in which a very minute degree of compressibility may be represented, is by supposing only a small proportion of the vortices present to be coreless. When such a medium is subjected to increased pressure, so as to diminish the volume occupied by the vacuous cores, the circulation around each vortex remaining unaltered, the energy of the turbulence will be increased by an amount equal to the work done by the pressure, and this additional energy is to be regarded as potential when the turbulently moving liquid is treated as a continuous medium.

74. If we suppose each core, whether vacuous or consisting of rotationally moving liquid, to be of very small diameter in relation to the radius of curvature of its "curved axis," and very small also in comparison with the distance between neighbouring vortices, an expression may readily be obtained for the compressional elasticity of the medium. If p is the mean pressure, we shall have, for any point in the liquid sufficiently remote from vortices,

$$p=\frac{\Omega_1^2\rho}{8\pi^2r_1^2}=\frac{\Omega_2^2\rho}{8\pi^2r_2^2}=\ldots; \quad . \quad . \quad . \quad (99)$$

where $r_1, r_2, \ldots$ are the radii of the various vacuous cores, $\Omega_1, \Omega_2, \ldots$ the circulations around them, and ρ the density of the liquid. Hence, extending the summation to the unit of volume, and calling $l_1, l_2, \ldots$ the lengths of the respective coreless vortices comprised in that volume,

$$\frac{\rho\Sigma l\Omega^2}{8\pi p}=\Sigma l\pi r^2=\text{U, say}; \quad . \quad . \quad . \quad . \quad (100)$$

so that U is the total volume of vacuous cores comprised in unit volume of the turbulent liquid. Also, assuming the l's to be invariable,

$$-\frac{dU}{dp}=\frac{\rho}{8\pi p^2}\Sigma l\Omega^2=\frac{8\pi^2}{\rho}\Sigma\frac{v}{\Omega^2}; \quad . \quad . \quad . \quad (101)$$

where, corresponding with any one vortex of strength Ω, there is a volume v of vacuous core comprised within the particular unit volume now considered. The left-hand member of (101) represents evidently the reciprocal of the bulk-modulus of elasticity appertaining to the medium as a whole, while the velocity $\mathfrak{V}$ of compressional waves is given by

$$\mathfrak{V}^2=\frac{1}{8\pi^2\Sigma(v/\Omega^2)}. \quad . \quad . \quad . \quad . \quad (102)$$

These expressions are given merely by way of illustration; they are of course far from being the most general, in the first place because of our assumption that all the vortices may be treated as linear, and in the second place because it has been supposed that the core of any one vortex either consists wholly of rotationally moving liquid or is wholly vacuous.

Discussion.

Mr C. S. Whitehead asked if the amount of energy stored in the æther was greater or less than it was in the past.

Prof. C. H. Lees asked if the æther considered was the old one or a new one. He referred to an interesting point in the Author's representation of gravitation, namely, that matter was produced by the absence of something from the æther. In the theory advocated by Osborne Reynolds matter was produced by a deficiency in the normal piling, and recent theories seek to explain matter and gravitation by the absence of something which is present in free æther.

Dr C. V. Burton, in reply to Mr Whitehead, said he was not aware whether the amount of energy in the æther varied with time or not.

XXVII. *On a Simple Physical Method of illustrating the Principles of Geometrical Optics.* *By* R. M. Archer, *A.R.C.Sc., B.Sc. (Lond.)**.

[Plate XXIV.]

The principles of geometrical optics are frequently illustrated by experiments in which the images of narrow obstacles are obscured by similar obstacles arranged in the line of vision. This method is somewhat tedious and is open to the objection that the path of a shadow is traced rather than that of a beam. The following method is free from these objections, and will be found interesting and convincing †.

The procedure is to allow light from a narrow rectilinear source to pass through a slit and fall upon a flat white surface at almost grazing incidence. In most cases the source may be the straight filament of a glow-lamp, and should be arranged parallel to the slit and not less than about 18 inches from it. Alternatively, a batswing burner with the flame "edge on" to the slit may be employed, but must be screened from draughts by a suitable enclosure.

It will be found easy to obtain upon the white surface a long narrow streak of light with sharp edges; and if a mirror be placed with its plane approximately normal to the surface another streak corresponding to the reflected ray will be seen. Similarly, the path of the beam after its emergence from a glass block or prism may be traced. When it is desired to trace the refracted beam before emergence, a glass tank of the required shape should be filled with water and the internal beam allowed to graze a white surface (*e. g.* that of a glazed tile) supported almost horizontally in the water. Provided, however, the glass block has a transparent base, all that is necessary is to coat the latter with white enamel paint and allow the internal beam

* Read May 8, 1908.

† My thanks are due to Mr. R. S. Bowman, B.Sc., for the valuable assistance he has rendered during the preparation of the photographs.

to strike this at a low angle. If in the above cases of refraction the incident light be bright, emergent beams which have been internally reflected several times will be observed.

Very beautiful effects can be obtained by using many slits and casting the beam from a distant optical lantern upon them. This mode of illumination was adopted in the case of the photographs reproduced (Pl. XXIV.). Perhaps the most striking case to select is that of a narrow concave strip cut from a cylindrical mirror. If this be illuminated by a very wide beam, a general convergent effect will be produced. The broad beam may then be analysed into many narrow ones by interposing a cardboard comb, and the formation of a caustic demonstrated. For this latter purpose it is better to skew the mirror until its main axis is oblique to the incident light. The broad beam may then be used and the skew caustic traced upon the surface by a black crayon. When the comb is interposed all the reflected rays will touch the curve, and the whole reflected set will sweep round it when the comb is given a transverse displacement. The best effects are obtained when the radiant is small, distant, and brilliant. In lecture demonstrations the prism or mirror may be fixed on a sheet of white cardboard attached to a vertical board.

When a divergent pencil is required a broad beam should be cast upon several narrow plane mirrors, and the latter turned until they converge the light upon the back of the slit. For laboratory purposes, however, it is generally sufficient to put the glow-lamp into different positions and trace the successive paths of the single beam.

Cylindrical lenses do not give such good effects as mirrors; but if they are allowed to project through a narrow aperture in the cardboard the results are fairly satisfactory.

Darkening the whole room renders the demonstrations more effective, but for individual work local screening is sufficient.

In conclusion it may be remarked that quantitative results can be obtained of an accuracy comparable with that usually reached in experiments with the ordinary optical bench.

Discussion.

Prof. C. H. Lees expressed his interest in the neatness of the experiments, and said the Author's method of dividing up a beam was a very useful one.

Mr A. Campbell congratulated the Author and remarked that the methods described could be applied to other optical experiments. In using a vibration galvanometer or an oscillograph, an easy way to obtain a curve of the wave-form was to put obliquely in the path of the beam a sheet of white paper and vibrate it.

Dr W. H. Eccles also congratulated the Author upon his paper.

XXVIII. *Note on the Amount of Water in a Cloud formed by Expansion of Moist Air.* *By* W. B. Morton, *Professor of Natural Philosophy, Queen's College, Belfast* *.

The calculation of the total mass of water condensed, when a volume of air saturated with vapour receives a given sudden expansion, is an important step in Prof. J. J. Thomson's determination of the charge on a gaseous ion †. In conjunction with the size of a single drop, as found from the rate of fall of the cloud, it gives the number of drops in unit volume and so the number of the ions which form the nuclei of the drops. In making the calculation Prof. Thomson, following C. T. R. Wilson, makes the assumption that the air is cooled down to the full extent by the adiabatic expansion before the drops begin to form. Then condensation proceeds and the latent heat liberated warms the air until the density of the remaining vapour is such as to saturate the air at the increased temperature. This process is, of course, irreversible. There is no reason to doubt that it represents closely the actual experimental conditions; but with a view to estimate the amount of error which would be introduced by a small departure from the assumed process, it seems of some interest to calculate, for comparison, the result of the other extreme

* Read June 12, 1908.

† 'Conduction of Electricity through Gases,' p. 122.

assumption, viz., that the expansion and condensation exactly keep pace with each other, the density of vapour at each stage being the saturation-density corresponding to the instantaneous value of the temperature. In this case the process would be reversible, and we can equate the initial and final values of the entropy of the combined mass of air and water-substance. Since an irreversible change is accompanied by an increase of entropy, it is easy to see that on the new assumption we shall arrive at a lower value for the density of the remaining vapour and higher value for the mass of water in the cloud.

In both cases the solution is most easily obtained by a graphical method. We plot a curve connecting the vapour density ρ and temperature t at any stage of the condensation, and find where it intersects the curve representing the maximum density of saturated vapour at different temperatures. In the irreversible case the former curve is the straight line.

$$\rho=\frac{\rho'}{x}-\frac{\mathrm{CM}}{\mathrm{L}}(t-t_2)^*;$$

where (using the notation of the authors quoted) ρ' is the initial saturation density, x the expansion ratio, t_2 the calculated temperature after adiabatic expansion, M the mass of unit volume of air after expansion, C specific heat of air at constant volume, L latent heat of the vapour. The heat-capacity of the drops is neglected in comparison with that of the air. For the reversible case the equation of constant entropy comes out

$$(x\mathrm{MC}+s\rho')\log\frac{273+t}{273+\mathrm{T}}+\mathrm{L}\left\{\frac{x\rho}{273+t}-\frac{\rho'}{273+\mathrm{T}}\right\}+x\mathrm{MR}\log x=0,$$

where, in addition to the symbols already define, s is the specific heat of water, T the temperature before expansion, and R the gas constant for one gram of air.

I have plotted this curve for the particular case discussed in detail by Prof. Thomson, viz., starting from an initial tem-

* Equation (2), *loc. cit.*

perature of 16° C., and giving an expansion of 1·36. The graph comes out practically a straight line running parallel to the irreversible line, both lines being nearly perpendicular to the saturation-density curve with which their intersections are sought. The numerical values come out on the reversible hypothesis,

$\rho = 4{\cdot}87 \times 10^{-6}$, $t = 0^{\circ}{\cdot}5$, mass of cloud per cm.^{3} $5{\cdot}06 \times 10^{-6}$grm.

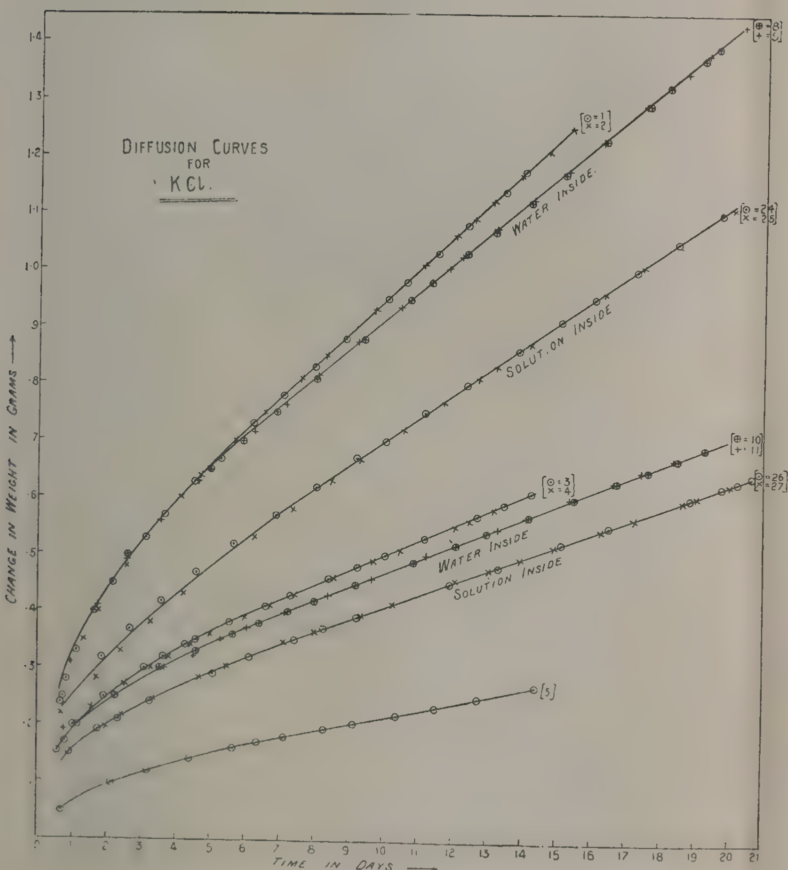
as compared with the values obtained on the accepted assumption by Prof. Thomson,

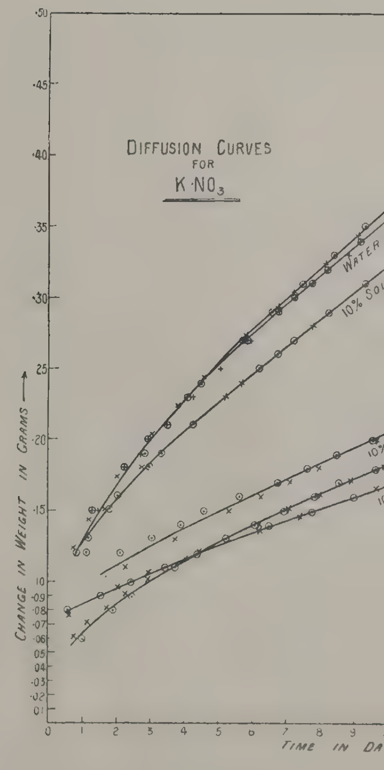
$\rho = 5{\cdot}15 \times 10^{-6}$, $t = 1^{\circ}{\cdot}2$, mass of cloud per cm.^{3} $4{\cdot}77 \times 10^{-6}$ grm.

So that even on the extreme assumption here made we arrive at a value differing by only 6 per cent. from the other. We can infer that no error, comparable with the other inaccuracies attached to the methods of measurement, will be involved if the assumption of complete adiabatic cooling of the air does not exactly correspond with the facts.

27th May, 1908.

FIG. 4.


DIFFUSION CURVES
FOR
KCl.
WATER INSIDE.
SOLUTION INSIDE
WATER INSIDE
SOLUTION INSIDE
CHANGE IN WEIGHT IN GRAMS
TIME IN DAYS


DIFFUSION CURVES
FOR
K·NO3
CHANGE IN WEIGHT IN GRAMS

. 5.

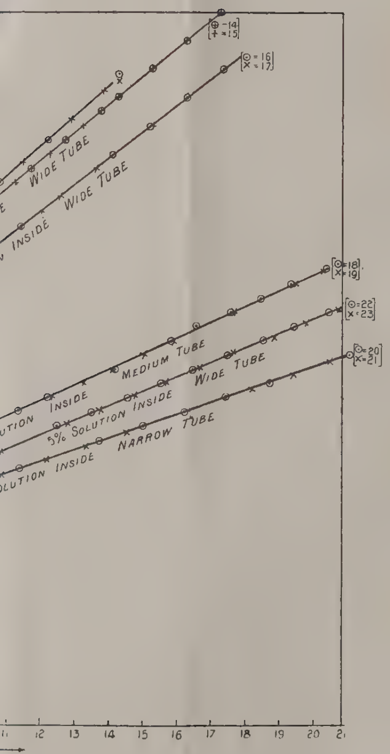

WIDE TUBE
WIDE TUBE
INSIDE
MEDIUM TUBE
INSIDE
5% SOLUTION INSIDE
WIDE TUBE
NARROW TUBE
INSIDE
12
13
14
15
16
17
18
19
20

FIG. 6.

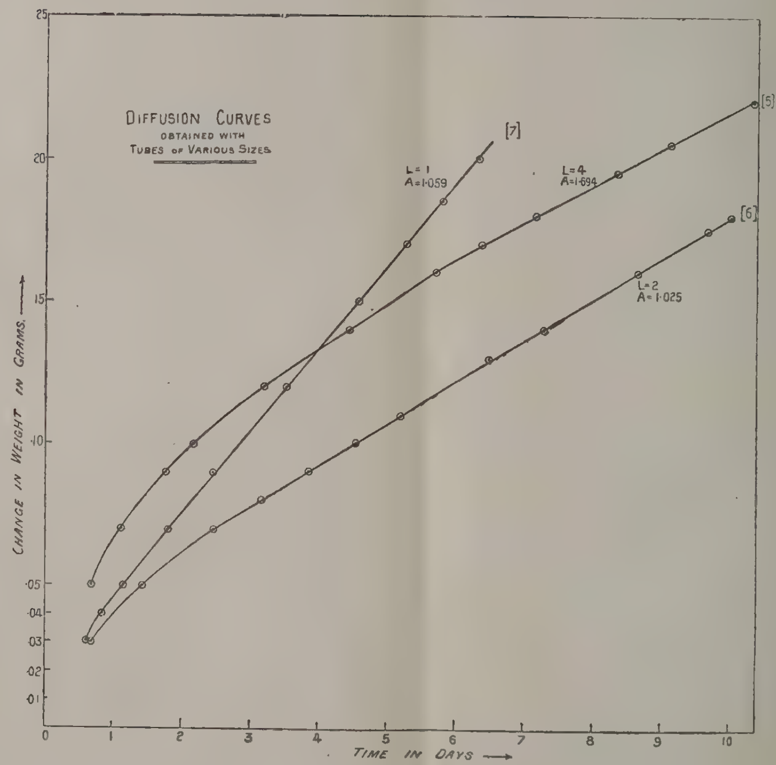
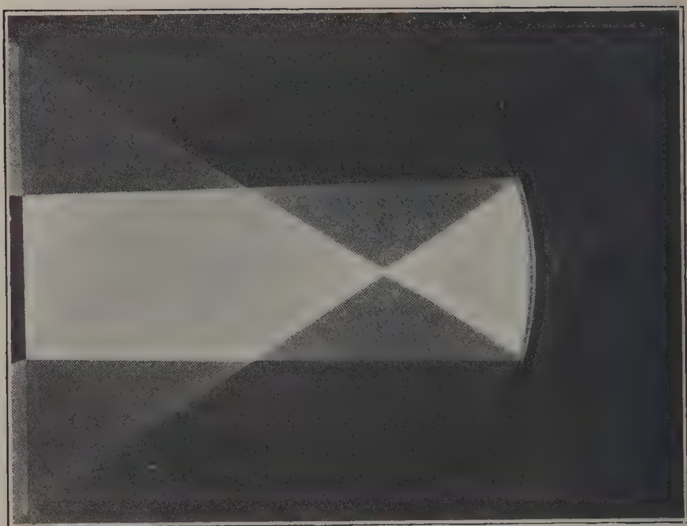
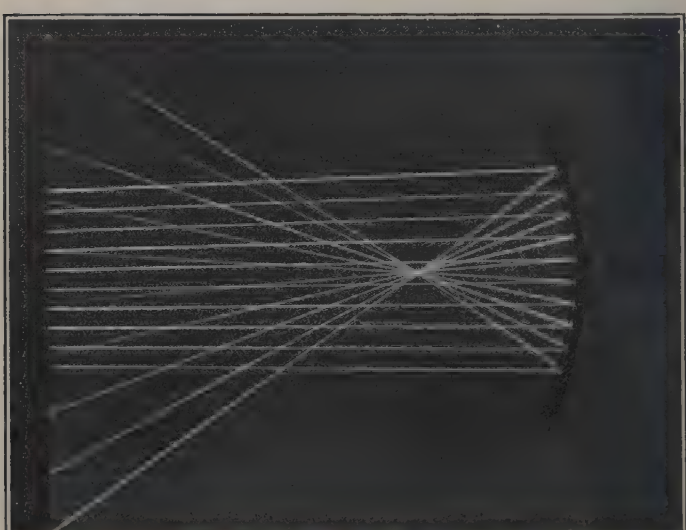

DIFFUSION CURVES
OBTAINED WITH
TUBES OF VARIOUS SIZES
L=1
A=1·059
L=4
A=1·694
L=2
A=1·025
[5]
[6]
[7]
CHANGE IN WEIGHT IN GRAMS.
TIME IN DAYS

Fig. 1.



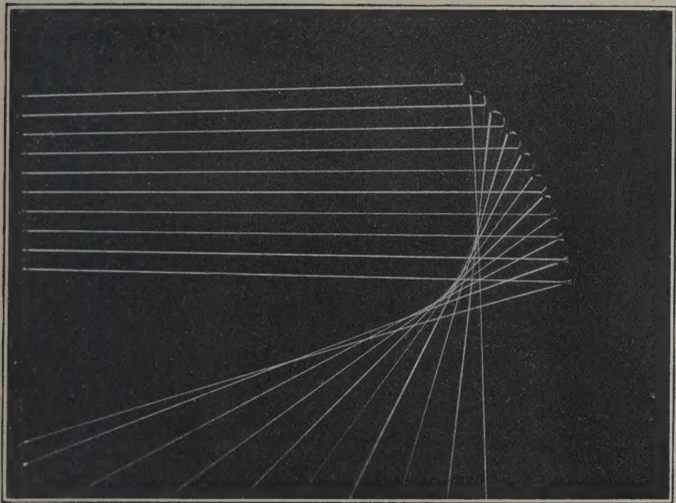
General convergent action of concave (cylindrical) mirror.

Fig. 2.



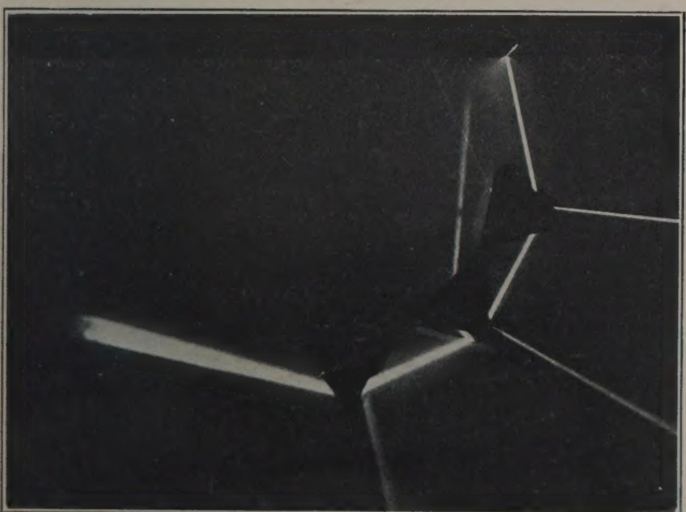
Analysis of fig. 1 into narrow beams.

Fig. 3.



Concave cylindrical mirror. Rays touching caustic curve.

Fig. 4.



Dispersion by train of prisms.

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